A Mechanics-Based Approach for Determining Deflections of Stacked Multi-Storey Wood-Based Shear Walls

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INTRODUCTION

The 2009 edition of CSA Standard O86, *Engineering Design in Wood* (CSA 2009), provides an equation for determining the deflection of shear walls. It is important to note that this equation only works for a single-storey shear wall with load applied at the top of the wall. While the equation captures the shear and flexural deformations of the shear wall, it does not account for moment at the top of the wall and the cumulative effect due to rotation at the bottom of the wall, which would be expected in a multi-storey structure.

In this fact sheet, a mechanics-based method for calculating deflection of a multi-storey wood-based shear wall is presented.

Deflections of Single-Storey Wood-Based Shear Walls

In CSA 086-09, the deflection of a single-storey shear wall can be determined as follows:

$$\Delta = \frac{2vH^3}{3EAL} + \frac{vH}{B_v} + 0.0025He_n + \frac{H}{L}d_a \quad [1]$$

In this equation, the first term is derived from the flexural deflection of a vertical cantilever beam with point load applied at the top of the beam (Figure 1):

$$\Delta_f = \frac{VH^3}{3EI} = \frac{vLH^3}{3E\left(\frac{AL^2}{2}\right)} = \frac{2vH^3}{3EAL} \quad [2]$$

where

V = point load at the top of shear wall

H = height of the shear wall

E = elastic modulus of boundary member

I = moment of inertia

A = cross-sectional area of the boundary member

L = length of shear wall

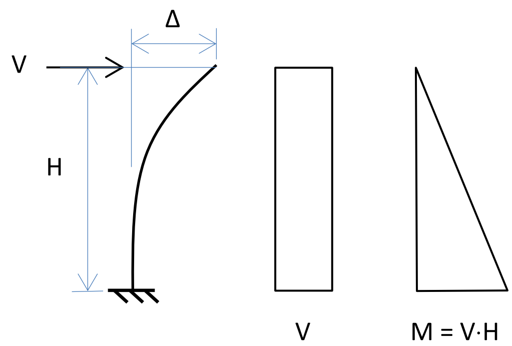


Figure 1. Shear and moment diagram of a single-storey shear wall.

Deflections of Stacked Multi-Storey Wood-Based Shear Walls

For a stacked multi-storey shear wall, the effect of moment at the top of the wall and the cumulative effect due to rotation at the bottom of the wall need to be taken into consideration. The deflection comprises five parts: the deflection due to bending, panel shear, nail slip, wall anchorage system elongation, and rotation at the bottom of the shear wall. The inter-storey deflection of the *i-th* storey, Δ_i , for a stacked multi-storey shear wall can be derived as follows:

$$\Delta_i = \Delta_{b,i} + \Delta_{s,i} + \Delta_{n,i} + \Delta_{a,i} + \Delta_{r,i} \qquad [3]$$

where

$\Delta_{b,i}$ = inter-storey deflection due to bending of the *i-th* storey

$\Delta_{s,i}$ = inter-storey deflection due to panel shear of the *i-th* storey

$\Delta_{n,i}$ = inter-storey deflection due to nail slip of the *i-th* storey

$\Delta_{a,i}$ = inter-storey deflection due to vertical elongation of the wall anchorage system of the *i-th* storey

$\Delta_{r,i}$ = inter-storey deflection due to rotation at the bottom of the shear wall of the *i-th* storey

1. Deflection due to bending, $\Delta_{b,i}$

Figure 2 shows the shear force and moment distribution in a multi-storey shear wall. Unlike single-storey shear walls where moment is zero at the top of the wall, there are also moments at the top of the walls in the multi-storey shear wall transferred from storeys above. Using the engineering mechanics, the flexural deflection for a shear wall at the *i-th* storey can be expressed as follows:

$$\Delta_{b,i} = \frac{V_i H_i^3}{3(EI)_i} + \frac{M_i H_i^2}{2(EI)_i} \qquad [4]$$

where

V_i = shear force of *i-th* storey

$$= \sum_{j=i}^{n} F_j$$

M_i = overturning moment at top of *i-th* storey

$$= \sum_{j=i+1}^{n} V_j H_j$$

H_i = height of the shear wall of *i-th* storey

$(EI)_i$ = bending stiffness of the shear wall of *i-th* storey

n = total number of storeys of the building

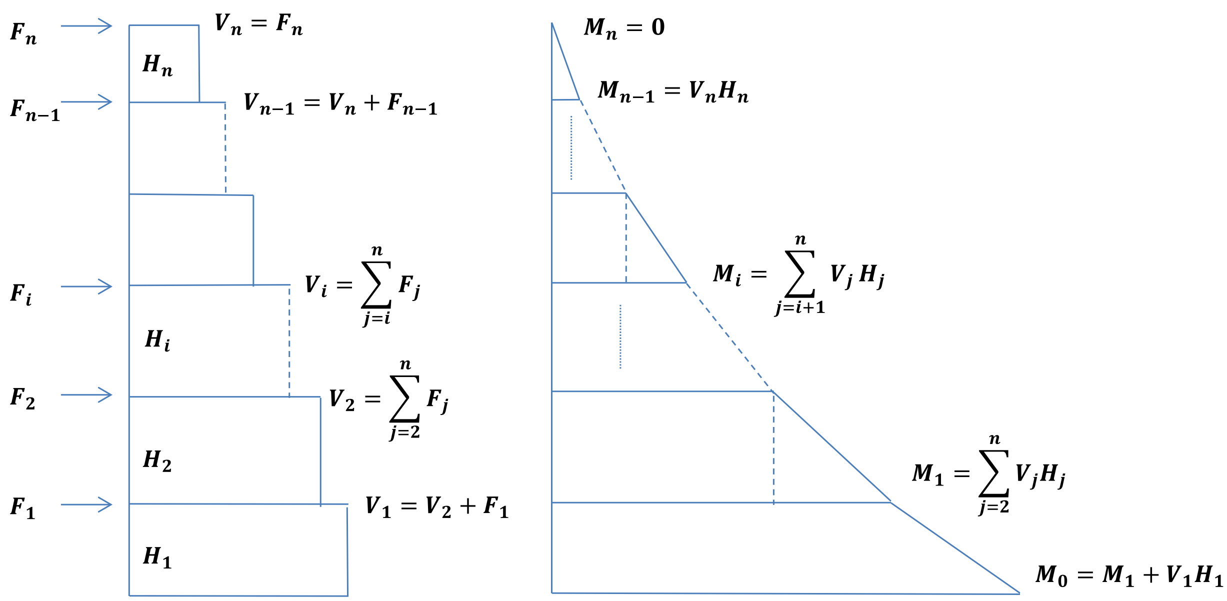


Figure 2. Shear and moment diagram of a stacked multi-storey shear wall.

In Equation 4, the moment of inertia, $I = \frac{AL^2}{2}$, applies to a shear wall where a discrete hold-down anchor system (Figure 3) is used and wood end studs at both ends of the wall are relied on to resist the overturning moment.

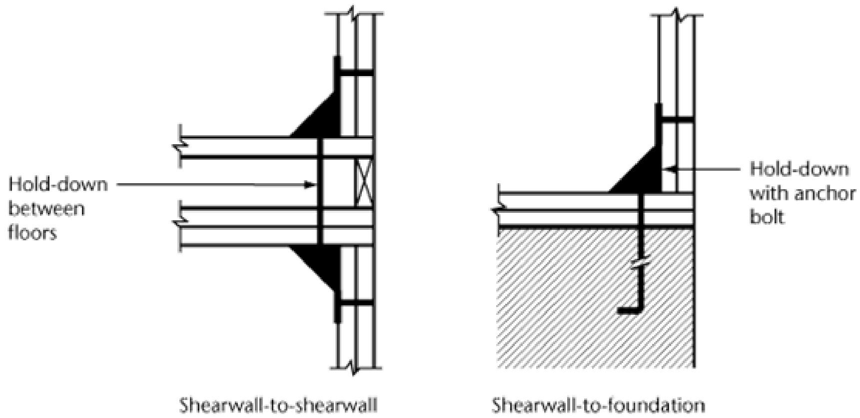


Figure 3. Discrete hold-downs in a shear wall.

Where a continuous rod in lieu of discrete hold-down is used in a shear wall, the tension and compression forces due to overturning moment will be resisted by the continuous steel rod and wood end studs (see Figure 4), respectively. As a result, the transformed bending stiffness of the shear wall, EI_{tr}, should be used in Equation 4. It can be obtained as follows:

$$E = E_c \qquad [5]$$

$$n = \frac{E_t}{E_c} \qquad [6]$$

$$A_{t,tr} = A_t \cdot n \qquad [7]$$

$$y_{tr} = \frac{A_c \cdot L_c}{A_{t,tr} + A_c} \qquad [8]$$

$$I_{tr} = A_{t,tr} \cdot y_{tr}^{\ 2} + A_c \cdot (L_c - y_{tr})^2 \qquad [9]$$

Where I_{tr} is the transformed moment of inertia of the shearwall and the tension chord area is transformed by $n = E_t / E_c$.

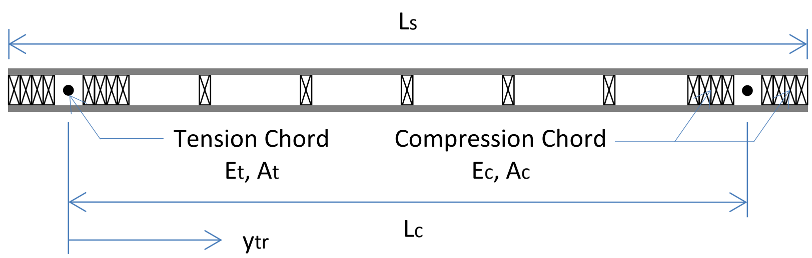


Figure 4. Shear wall section.

2. Deflection due to panel shear, $\Delta_{s,i}$

The deflection due to panel shear can be obtained as follows:

$$\Delta_{s,i} = \frac{V_i H_i}{L_i B_{v,i}} \qquad [10]$$

where

L_i = length of the shear wall of *i-th* storey

$B_{v,i}$ = shear-through-thickness rigidity of the sheathing of *i-th* storey

3. Deflection due to nail slip, $\Delta_{n,i}$

The deflection due to nail slip can be obtained as follows:

$$\Delta_{n,i} = 0.0025 H_i e_{n,i} \qquad [11]$$

where

$e_{n,i-1}$ = nail deformation of *i-th* storey

4. Deflection due to vertical elongation of the wall anchorage system, $\Delta_{a,i}$

The deflection due to vertical elongation of the wall anchorage system can be obtained as follows:

$$\Delta_{a,i} = \frac{H_i}{L_i} d_{a,i} \qquad [12]$$

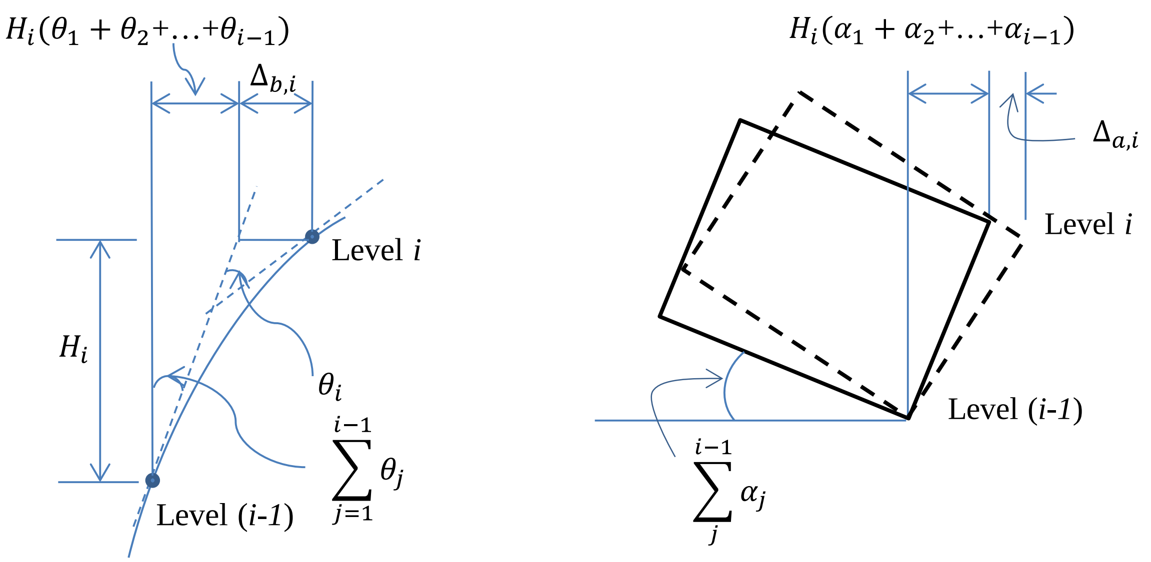
where

$d_{a,i}$ = total vertical elongation of the wall anchorage system of *i*-th storey

5. Deflection due to rotation at the bottom of the wall, $\Delta_{r,i}$

For a shear wall at *i*-th storey of a multi-storey, both the cumulative rotation due to bending, $\sum_{j=1}^{i-1}\theta_j$, and the cumulative rotation due to wall anchorage system elongation, $\sum_{j=1}^{i-1}\alpha_j$, from the storeys below contribute to the rotation of the shear wall at *i-th* storey, which in turn affects the inter-storey deflection of the wall at *i-th* storey. Figure 5 shows the effect of the cumulative rotation due to bending and the effect of the cumulative rotation due to wall anchorage system elongation. Assuming rotations between the adjacent storeys are continuous, the inter-storey deflection due to the rotation at the bottom of *i-th* storey can be written as:

$$\Delta_{r,i} = H_i\sum_{j=1}^{i-1}\theta_j + H_i\sum_{j=1}^{i-1}\alpha_j = H_i\sum_{j=1}^{i-1}\left(\frac{M_jH_j}{(EI)_j} + \frac{V_jH_j^{\,2}}{2(EI)_j}\right) + H_i\sum_{j=1}^{i-1}\frac{d_{a,j}}{L_j} \quad [13]$$



a) cumulative rotation due to bending

b) cumulative rotation due to wall anchorage system elongation

Figure 5. Deflection due to rotation at the bottom of the wall.

In platform wood-frame construction, the stacked shear walls are separated by diaphragm at each storey. This discontinuity has a direct impact on the rotation of a stacked multi–shear wall at the bottom of *i-th* storey. If the diaphragm is flexible in the out-of-plane direction, the diaphragm will likely rotate the same amount as the shear wall below. In this case, it can be assumed that the shear wall at the bottom of *i-th* storey will have the same rotation as that at the top of *(i-1)-th* storey, and the amount of rotation from the storey below can be determined by Equation 13. On the other hand, if the diaphragm is rigid in the out-of-plane direction, it is unlikely that the diaphragm will rotate the same amount as the shear wall below. In this case, the rotation of shear wall due to storeys below can be ignored. In design, it is the judgment of design engineers to decide how much the rotation due to lower storeys should be taken into consideration.

By taking into consideration the rotation of the storeys below, this method should provide a conservative approach for determining deflections and inter-storey drifts. On the other hand, this approach may underestimate the shear wall stiffness and overestimate the building period, and therefore potentially underestimate the base shear the building may experience during an earthquake if one is to use a period determined by a mechanics-based method other than the Code-based period. However, this may be addressed by the 2 × period cut-off imposed by the Code for base shear calculation. A comparative analysis conducted by the Canadian Wood Council looked at different methods for deflection calculation, including the mechanics-based approach which takes into account the rotation from the storeys below and the approach which excludes the rotation. This study showed that the periods calculated using both methods were significantly higher than 2 × Code-based period for the design example, and suggested that using 2 × the Code period is a reasonable assumption for the initial design. However, it will be important that designers confirm that this is a reasonable assumption by calculating the period. It is recommended in this study that the mechanics-based approach (taking into account the rotational effects) be used for determining drifts, stiffnesses, and building periods for wood-based shear walls. This should lead to a reasonable distribution of forces for a seismic force resisting system of wood-based shear walls and should provide a conservative approach for determining deflections and drifts.

References

Canadian Standards Association (CSA). 2009. CAN/CSA O86-09 Engineering design in wood. Rexdale, Ont.

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