



Linear Dynamic Analysis for Wood-based Shear Walls & Podium Structures

*Part 1: Developing Input Parameters for
Linear Dynamic Analysis*



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INTRODUCTION

Utilizing Linear Dynamic Analysis (LDA) for designing steel and concrete structures has been common practice over the last 25 years.

Utilizing Linear Dynamic Analysis (LDA) for designing steel and concrete structures has been common practice over the last 25 years. Once preliminary member sizes have been determined for either steel or concrete, building a model for LDA is generally easy as the member sizes and appropriate stiffness can be easily input into any analysis program. However, performing an LDA for a conventional wood-frame structure has been, until recently, essentially non-existent in practice. The biggest challenge is that the stiffness properties required to perform an LDA for a wood-based system are not as easily determined as they are for concrete or steel structures. This is mostly due to the complexities associated with determining the initial parameters required to perform the analysis.

With the height limit for combustible construction limited to four stories under the National Building Code of Canada, it was uncommon for designers to perform detailed analysis to determine the stiffness of shear walls, distribution of forces, deflections, and inter-storey drifts. It was only in rare situations where one may have opted to check building deflections. With the recent change in allowable building heights for combustible buildings from four to six storeys under an amendment to the 2006 BC Building Code, it has become even more important that designers consider more sophisticated methods for the analysis and design of wood-based shear walls. As height limits increase, engineers should also be more concerned with the assumptions made in determining the relative stiffness of walls, distribution of forces, deflections, and inter-storey drifts to ensure that a building is properly detailed to meet the minimum Code objectives.

Although the use of LDA has not been common practice, the more rigorous analysis, as demonstrated in the APEGBC bulletin on 5- and 6-storey wood-frame residential building projects (APEGBC 2011), could be considered the next step which allows one to perform an LDA. This fact sheet provides a method to assist designers who may want to consider an LDA for analyzing wood-frame structures. It is important to note that while LDA may provide useful information as well as streamline the design of wood-frame structures, it most often will not be necessary. However, designers may consider using LDA for the following reasons:

- Consider the effect of higher mode participation on force distributions and deflections.
- Better determine building deflections and floor drifts.
- Allow for three-dimensional modelling.
- Reduce the minimum Code torsional effect required under the equivalent static design.
- Better consider the effect of podium structures (vertical changes in $R_d R_o$).
- Compare the stiffness of various shear wall systems where mixed systems are used.

Proposed Method for Linear Dynamic Analysis

Although there may be different methods available to designers, the following steps could be used as a rational approach for performing LDA for wood structures. These are similar to the common practice for steel and concrete, and can be carried out in a design office using readily available commercial software.

Step 1 – Initial Analysis and Design

The first step requires performing an initial analysis and design such that the properties of each wall forming part of the lateral system can be determined. This will allow design engineers to determine the necessary information required to determine a shear wall's stiffness and deflection characteristics.

Below is a recommended method for the initial design:

1. Determine the building seismic forces at each level using the equivalent static force procedure in the National Building Code of Canada and provincial building codes, provided the building can be classified as regular and conforms to code requirements that would otherwise prohibit using the equivalent static approach.

The seismic forces can be determined by using either the building period T_a (based on the imperial formula in the Code) or period T determined by mechanical methods, with an upper limit of $2T_a$. If period T is used, the base shear V needs to be increased by a factor of 1.2 in accordance with the 2012 BC Building Code. The assumed building period must be verified either by mechanics-based methods or by running an LDA.

2. Determine the initial distribution of forces to each wall based on an assumed distribution, such that each wall may be designed to the required force level. Initial assumptions could include assuming the diaphragm to be flexible, rigid with the stiffness of each wall being assumed proportional to the wall length, or a combination of the two methods. Regardless, the minimum design forces at this stage will need to be re-adjusted if the forces are found to be higher once the LDA has been performed.

Step 2 – Determination of Input Parameters for Performing an LDA

This step utilizes the information developed from the preliminary analysis determined in Step 1 combined with proposed modifications to the 4-part equation for deflection provided in CSA 086-09, in order that representative input values can be determined for use in an LDA for a multi-level structure. Modifications to the 4-part equation include developing input parameters that replicate both the bending stiffness and shear stiffness while accounting for non-linear nail slip and softening effects due to hold-down slip, anchor shortening, and plate crushing. Shrinkage is separated from the equation and treated independently.

The following section proposes a method for determining input parameters upon which a Linear Dynamic Analysis can be performed for a wood-based shear wall system with most commercial software.

Mechanical Properties of Shear Walls for LDA Analysis

Figure 1 shows a generic cross-section of a wood-frame shear wall. The basic wood shear wall section includes the following components:

- Chord members (in this case, wood posts in compression and steel rods in tension)
- Sheathing (generally plywood or orientated strand board) on one or two sides
- Nailing (perimeter nail spacing / interior nailing)

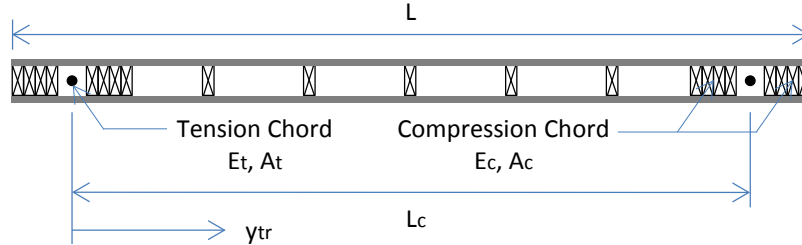


Figure 1. Shear wall section.

These three variables often vary by storey in a multi-storey wood-frame building and are the main variables from which a wall's stiffness can be determined. This is the reason that an initial analysis must be performed to determine the wall characteristics at each floor.

Using the mechanics-based approach (Newfield et al. 2013), the inter-storey deflection of a stacked multi-storey shear wall at i -th storey can be determined below:

$$\Delta_i = \Delta_{b,i} + \Delta_{s,i} + \Delta_{n,i} + \Delta_{a,i} + \Delta_{r,i} \quad [1]$$

$$\Delta_{b,i} = \frac{V_i \cdot H_i^3}{3(EI)_i} + \frac{M_i \cdot H_i^2}{2(EI)_i} \quad [2]$$

$$\Delta_{s,i} = \frac{V_i \cdot H_i}{L_i \cdot B_{v,i}} \quad [3]$$

$$\Delta_{n,i} = 0.0025 H_i \cdot e_{n,i} \quad [4]$$

$$\Delta_{a,i} = \frac{H_i}{L_i} d_{a,i} \quad [5]$$

$$\Delta_{r,i} = H_i \sum_{j=1}^{i-1} \theta_j + H_i \sum_{j=1}^{i-1} \alpha_j = H_i \sum_{j=1}^{i-1} \left(\frac{M_j H_j}{(EI)_i} + \frac{V_j H_j^2}{2(EI)_j} \right) + H_i \sum_{j=1}^{i-1} \frac{d_{a,j}}{L_j} \quad [6]$$

where

Δ_i = inter-storey deflection at *i-th* storey, mm

$\Delta_{b,i}$ = deflection at *i-th* storey due to bending, mm

$\Delta_{s,i}$ = deflection at *i-th* storey due to panel shear, mm

$\Delta_{n,i}$ = deflection at *i-th* storey due to nail slip, mm

$\Delta_{a,i}$ = deflection at *i-th* storey due to wood plate bearing and vertical elongation of anchorage system, mm

$\Delta_{r,i}$ = deflection at *i-th* storey due to rotation at the bottom of the shear wall, mm

V_i = shear force at level *i*, N

$$= \sum_{j=1}^n F_j$$

M_i = overturning moment at level *i*, N.mm

$$= \sum_{j=i+1}^n V_j H_j$$

H_i = height of shear wall at *i-th* storey, mm

$(EI)_i$ = bending stiffness of shear wall at *i-th* storey, N/mm²

L_i = length of shear wall at *i-th* storey, m

$B_{v,i}$ = shear-through-thickness rigidity of wall panels at *i-th* storey, N/mm

$e_{n,i}$ = nail deformation for shear wall at *i-th* storey, mm

$d_{a,i}$ = sum of vertical deformation at *i-th* storey due to wood plate bearing and anchorage slip of shear wall, mm

For the purpose of LDA, the method for determining physical properties of a shear wall in terms of t_{eq} , L , E , and G , which can be easily represented as a standard beam in most commercial software, are provided below.

Flexural Deformation ($\Delta_b + \Delta_a$)

As indicated in Equation 2, the component, Δ_b , is simply related to a shear wall's bending stiffness at each storey. It can be represented in commercial software by bending stiffness EI . Based on Figure 1, the basic input values for a shear wall can be determined as follows:

$$b = L \tag{7}$$

$$t_{eq} = \frac{12I}{L^3} \tag{8}$$

$$E = E_c \tag{9}$$

For shear walls with discrete hold-downs (as shown in Figure 2), wood end studs at both ends of the wall provide the resistance to the overturning moment. Therefore, the moment of inertia can be obtained as follow:

$$I = \frac{AL_c^2}{2} \quad [10]$$

where A is the cross section of end studs and L_c is the distance between the centres of the end studs

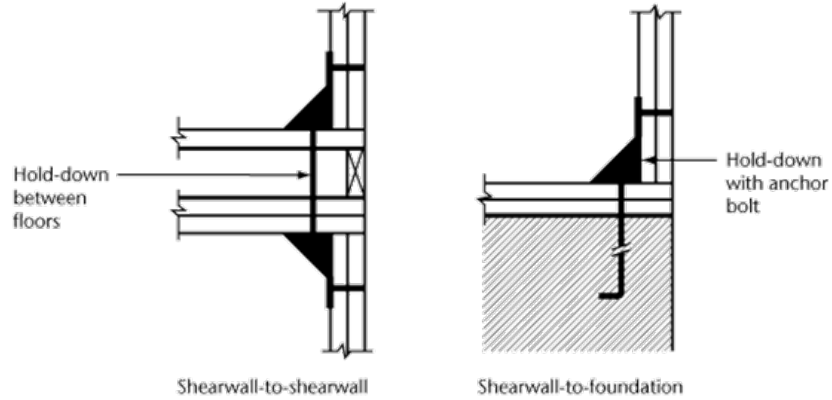


Figure 2. Discrete hold-downs in a shear wall.

Where a continuous rod in lieu of discrete hold-down is used in a shear wall, the tension and compression forces due to overturning moment will be resisted by the continuous steel rod and wood end studs (as shown in Figure 1), respectively. As a result, the transformed bending stiffness of the shear wall, EI_{tr} , should be used in Equation 8. It can be obtained as follows:

$$n = \frac{E_t}{E_c} \quad [11]$$

$$A_{t,tr} = A_t \cdot n \quad [12]$$

$$y_{tr} = \frac{A_c \cdot L_c}{A_{t,tr} + A_c} \quad [13]$$

$$I = A_{t,tr} \cdot y_{tr}^2 + A_c \cdot (L_c - y_{tr})^2 \quad [14]$$

The above input parameters need to be further adjusted to take into consideration the component Δ_a , which is caused by two components: 1) the bearing deformation due to wood plate in compression, and 2) anchorage slip in tension. This can be done by adjusting the modulus of elasticity of shear wall boundary members in compression and tension. For boundary members in compression, the total deformation of the boundary members can be obtained as follows:

$$\Delta_{compression} = d_c + d_{a,c} = \frac{T_c \cdot H_{//}}{E_{c//} A_c} + \frac{T_c \cdot H_{\perp}}{E_{c\perp} A_c} \quad [15]$$

where:

T_c = compression load

$E_{c//}$ = modulus of elasticity parallel to grain

$E_{c\perp}$ = modulus of elasticity perpendicular to grain

$H_{//}$ = length of boundary member with grain parallel to the applied load (end studs)

H_{\perp} = length of boundary member with grain perpendicular to the applied load (top and bottom plates)

A_c = cross section of boundary member in compression

Expressing $\Delta_{compression}$ as a function of $E_{c,eq}$, Equation 15 can be rewritten as:

$$\Delta_{compression} = \frac{T_c \cdot H}{E_{c,eq} \cdot A_c} = \frac{T_c \cdot H_{//}}{E_{c//} A_c} + \frac{T_c \cdot H_{\perp}}{E_{c\perp} A_c} \quad [16]$$

Therefore, $E_{c,eq}$ can be written as:

$$E_{c,eq} = \frac{H \cdot E_{c\perp} \cdot E_{c//}}{H_{//} \cdot E_{c\perp} + H_{\perp} \cdot E_{c//}} = \frac{(H_{//} + H_{\perp}) E_{c\perp} E_{c//}}{H_{//} \cdot E_{c\perp} + H_{\perp} \cdot E_{c//}} \quad [17]$$

Similarly for boundary members in tension, the total deformation of the boundary members consists of elongation of the tension chord, elongation of the anchorage system including anchor slip, rod elongation, and plate crushing. In order to take into consideration the additional elongation of the anchorage system, the E_t can be replaced by $E_{t,eq}$. Assuming the additional elongation is proportional to the applied load, the total elongation of chord member in tension can be expressed as follows:

$$\Delta_{tension} = d_t + d_{a,t} = \frac{T_f \cdot H_s}{E_t \cdot A_t} + \frac{T_f}{T_r} d_{max} \quad [18]$$

where:

d_t = elongation of chord member in tension over its length;

d_{max} = maximum anchorage slip (for discrete hold-downs) or maximum plate crushing (for continuous steel rod) at anchorage capacity. The value of d_{max} is generally available from test data and is often published within product catalogues.

E_t = modulus of elasticity, which is the modulus of elasticity of end studs for discrete hold-downs or the modulus of elasticity of steel rod for continuous steel rod

A_t = cross section of boundary member in tension

T_f = tension load

T_r = anchorage capacity

Expressing $\Delta_{tension}$ as a function of $E_{t,eq}$, Equation 18 can be rewritten as:

$$\Delta_{tension} = \frac{T_f \cdot H_s}{E_{t,eq} \cdot A_t} = \frac{T_f \cdot H_s}{E_t \cdot A_t} + \frac{T_f}{T_r} d_{max} \quad [19]$$

Therefore, $E_{t,eq}$ can be written as:

$$E_{t,eq} = \frac{H_s}{\left(\frac{H_s}{E_t} + \frac{A_t}{T_r} d_{max} \right)} \quad [20]$$

By using $E_{c,eq}$ and $E_{t,eq}$ in place of E_c and E_t in Equation 11, the effects of elongation due to bearing and anchorage system are now accounted for in Equations 7, 8, and 9 and can represent the flexural stiffness required for an LDA. Each of these equations can easily be incorporated into a spreadsheet as part of the preliminary design such that the required input values for an LDA can quickly be determined.

Shear Deformation ($\Delta_s + \Delta_n$)

The shear deformation of a wood shear wall as shown in Figure 3 is related to two components. The first component, Δ_s , accounts for the linear shear deformation of the plywood panel. The second component, Δ_n , accounts for the shear deformation due to nail slip. As the nail slip is not linear and is often a large contributor to the overall deformations, iteration is required until the shear force on the nail is converged.

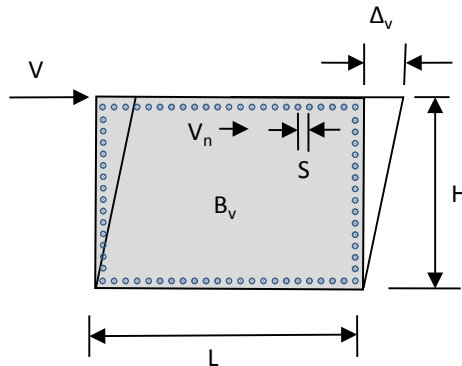


Figure 3. Shear deformation, Δ_v .

Based on established beam theory, the shear deformation for a beam can be calculated as follows:

$$\Delta_v = \frac{1.2 \cdot V \cdot H}{G_p \cdot A} = \frac{1.2 \cdot V \cdot H}{G_p \cdot L \cdot t_p} \quad [21]$$

where

G_p = shear modulus used in LDA

t_p = plywood panel thickness

By equating Equation 21 to Equations 3 and 4, it results in the following:

$$\frac{1.2 \cdot V \cdot H}{G_p \cdot L \cdot t_p} = \frac{V \cdot H}{L \cdot B_v} + 0.0025 H e_n \quad [22]$$

The corresponding shear per nail, V_n can be calculated as follows:

$$V_n = \frac{V}{L} s \quad [23]$$

Equation 23 can be rewritten as follows:

$$V = \frac{V_n \cdot L}{s} \quad [24]$$

Substituting Equation 24 into Equation 22, the G_p can be obtained as follow:

$$G_p = \frac{1.2}{\left(\frac{t_p}{B_v} + 0.0025 \frac{e_n \cdot s \cdot t_p}{V_n} \right)} \quad [25]$$

The value of G_p used in LDA can be further adjusted to the input thickness, t_{eq} , by multiplying Equation 25 with t_p / t_{eq} . Therefore, it becomes:

$$G_p = \frac{1.2}{\left(\frac{t_p}{B_v} + 0.0025 \frac{e_n \cdot s \cdot t_p}{V_n} \right)} \times \frac{t_p}{t_{eq}} = \frac{1.2}{t_{eq} \left(\frac{1}{B_v} + 0.0025 \frac{e_n \cdot s}{V_n} \right)} \quad [26]$$

To summarize, once a preliminary design has been performed to establish the properties of a shear wall including hold-downs, nailing, and panel thickness for each storey (Figure 4), the following equations can be used to determine both the flexural and shear stiffness for use in a linear dynamic analysis.

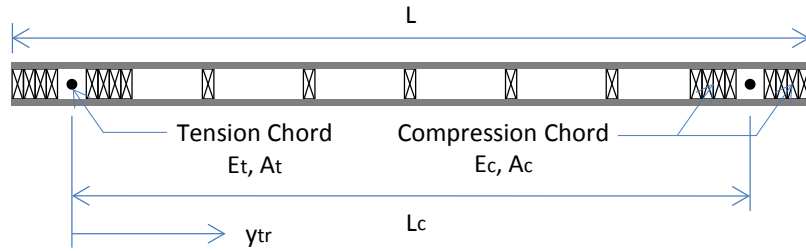


Figure 4. Shear wall section.

Transformed Properties

$$E_{c,eq} = \frac{H \cdot E_{c\perp} \cdot E_{c\parallel}}{H_{\parallel} \cdot E_{c\perp} + H_{\perp} \cdot E_{c\parallel}} = \frac{(H_{\parallel} + H_{\perp})E_{c\perp}E_{c\parallel}}{H_{\parallel} \cdot E_{c\perp} + H_{\perp} \cdot E_{c\parallel}}$$

$$E_{t,eq} = \frac{H}{\left(\frac{H}{E_t} + \frac{A_t}{T_r} d_{\max} \right)}$$

$$n = \frac{E_{t,eq}}{E_{c,eq}}$$

$$A_{t,tr} = A_t \cdot n$$

$$y_{tr} = \frac{A_c \cdot L_c}{A_{t,tr} + A_c}$$

$$I_{tr} = A_{t,tr} \cdot y_{tr}^2 + A_c \cdot (L_c - y_{tr})^2$$

Program Input Values

$$b = L$$

$$t_{eqi} = \frac{12I_{tr}}{L^3}$$

$$E = E_{c,eq}$$

$$G_p = \frac{1.2}{\left(\frac{t_p}{B_v} + 0.0025 \frac{e_n \cdot s \cdot t_p}{V_n} \right)} \times \frac{t_p}{t_{eq}} = \frac{1.2}{t_{eq} \left(\frac{1}{B_v} + 0.0025 \frac{e_n \cdot s}{V_n} \right)}$$

Deformation Due to Shrinkage

Wood shrinkage due to change of moisture content can cause gaps in the anchorage system which can lead to further shear wall deflections. If shrinkage compensators are not used, the gap created in the anchorage system as a result of shrinkage will cause the shear wall to rotate before the shear wall is engaged to resist the loads. In this case, the total building drift should be the deflection caused by the lateral load plus the additional deflections due to the shrinkage. The shear wall deflection due to shrinkage can be represented as follows:

$$\Delta_{sh,i} = \frac{H_i}{L_i} d_{sh,i} + \theta_{sh,i-1} \cdot H_i \quad [27]$$

where

$\Delta_{sh,i}$ = total deflection due to wood shrinkage at level i , m

H_i = height of shearwall segment at level i , m

L_i = length of shearwall segment at level i , m

$d_{sh,i}$ = gap created in anchorage system due to shrinkage at level i , m

$\theta_{sh,i}$ = rotation of shear wall at level i due to the summation of rotation of storeys below caused by shrinkage

As shrinkage can have a significant effect in the overall building deformations, it is recommended that shrinkage compensators be used for three-storey and taller wood-frame buildings, so that the deflection due to shrinkage can be neglected.

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