

## CWC Wood Engineering Assignment 1 Solutions

### QUESTION 1

#### Part A)

Sapwood:

- Is the living portion of the tree
- Function: Transports water/nutrients up the tree
- Not durable to decay

Heartwood:

- Is the dead portion of the tree (has dead cells)
- Function: Provides mechanical support (or structural strength) to the tree
- Resistance to decay

#### Part B)

Longitudinal axis: Parallel to the grain direction or to the direction of growth.

Radial axis: Perpendicular to the grain or radially outwards from the center of the tree.

Tangential axis: Tangent to the growth rings

**Part C)** Engineering wood consists of products manufactured by bonding together wood strands, veneers, lumber and fibers with glue/adhesives to form composite materials

Two advantages:

- Large panels can be manufactured from small diameter trees unsuitable for solid lumber
- Small pieces of wood (waste) and wood with defects can be used to build engineered wood. (i.e. particle and fiber-based boards)
- Prefabricated construction
- Architectural aesthetics
- Types of engineered wood:
  - Glulam
  - Plywood
  - Oriented Strandboard (OSB) and Waferboard
  - Parallel Strand Lumber (PSL)
  - Laminated Veneer Lumber (LVL)
  - Prefabricated Wood I-Joists
  - Cross-laminated timber (CLT)

#### Part D)

$$W_{\text{oven-dry}} = 20 \text{ m}^3 * 0.4 * 1000 \text{ kg/m}^3 = \mathbf{8000 \text{ kg}}$$

Therefore the oven dry weight is 8000 kg.

## QUESTION 2

### Part A)

The changes in dimensions are due to the reduction of moisture below the FSP. From Figure 1 the percentage of shrinkage due the changes of moisture from 28% to 7% are as follows: tangential = 6 %, radial = 3.1 %, and longitudinal = 0.23 %. The new dimensions will be:

$$\text{Shrinkage (S)} = \text{Original Dimension (D)} \times (M_i - M_F) \times [\% \text{Shrinkage}(M_F) / M_i]$$

#### Tangential:

$$S = 38 \text{ mm} \times (28 - 7) \times [0.06 \div 28] = 1.71 \text{ mm}$$

$$\text{Change in dimension} = 38 \text{ mm} - 1.71 \text{ mm} = 36.29 \text{ mm (4.5\% change)}$$

#### Radial

$$S = 89 \text{ mm} \times (28 - 7) \times [0.031 \div 28] = 2.06 \text{ mm}$$

$$\text{Change in dimension} = 89 \text{ mm} - 2.06 \text{ mm} = 86.94 \text{ mm (2.3\% change)}$$

#### Longitudinal

$$S = 2438 \text{ mm} \times (28 - 7) \times [0.0023 \div 28] = 4.2 \text{ mm}$$

$$\text{Change in dimension} = 2438 \text{ mm} - 4.2 \text{ mm} = 2434 \text{ mm (0.2\% change)}$$

### Part B)

#### Tangential:

$$S = 38 \text{ mm} \times (28 - 7) \times 0.002 = 1.60 \text{ mm}$$

$$\text{Change in dimension} = 38 \text{ mm} - 1.6 \text{ mm} = 36.4 \text{ mm (4.2\% change)}$$

This is compared to a 4.5% change in Part a)

#### Radial

$$S = 89 \text{ mm} \times (28 - 7) \times 0.002 = 3.74 \text{ mm}$$

$$\text{Change in dimension} = 89 \text{ mm} - 3.7 \text{ mm} = 85.3 \text{ mm (4.2\% change)}$$

This is compared to a 2.3% change in Part a)

#### Longitudinal

$$S = 2438 \text{ mm} \times (28 - 7) \times 0.00005 = 2.56 \text{ mm}$$

$$\text{Change in dimension} = 2438 \text{ mm} - 2.6 \text{ mm} = 2435.4 \text{ mm (0.1\% change)}$$

This is compared to a 0.2% change in Part a)

The equation given in O86-14 replicates the tangential and longitudinal shrinkage fairly well and overestimates the radial shrinkage. As shrinkage is most significant in the tangential direction, the decision to simplify the equation to purely parallel/perpendicular while conservatively favoring the tangential direction with the 0.002 coefficient is useful for design purposes as it provides a worst-case scenario for the perpendicular direction while preserving the relatively small longitudinal shrinkage. This is especially



useful for wood products comprised of many small wood plies/laminates as it eliminates the need to specifically determine the tangential/radial directions for perpendicular to grain shrinkage.

### QUESTION 3

Here we are interested in only the perpendicular to grain expansion to assess the clearance distance between members. Remember that only shrinkage/swelling will only occur up until the fibre saturation point of 28%. Using the O86 equation for shrinkage yields the following.

#### Beam:

Since the beam-column connection partially restrains the beam at its mid-depth, we can assume that the bottom third of the beam will swell downward, the upper third will swell upward, and the middle third will remain constant (although internal forces will develop – this is beyond the scope of this example). Therefore we will calculate the swelling based on 1/3 the total beam depth.

$$\begin{aligned} S_{\text{beam}} &= D \times (M_i - M_f) \times C_{\text{perpendicular}} \\ S_{\text{beam}} &= (1/3 \times 798 \text{ mm}) \times (5\% - 28\%) \times (0.002) \\ S_{\text{beam}} &= -12.2 \text{ mm (negative shrinkage - ie 12.2 mm of swelling)} \end{aligned}$$

#### Joist:

Since the joist is supported in bearing, the bearing contact will be maintained and all swelling will occur (upwards). Therefore the full joist depth is used in the calculation.

$$\begin{aligned} S_{\text{joist}} &= D \times (M_i - M_f) \times C_{\text{perpendicular}} \\ S_{\text{joist}} &= (380 \text{ mm}) \times (5\% - 28\%) \times (0.002) \\ S_{\text{joist}} &= -17.5 \text{ mm (negative shrinkage - ie 17.5 mm of swelling)} \end{aligned}$$

#### CLT:

Similarly to the joist, the CLT is supported in bearing and therefore the full depth is active in the swelling.

$$\begin{aligned} S_{\text{CLT}} &= D \times (M_i - M_f) \times C_{\text{perpendicular}} \\ S_{\text{CLT}} &= (105 \text{ mm}) \times (5\% - 28\%) \times (0.002) \\ S_{\text{CLT}} &= -4.8 \text{ mm (negative shrinkage - ie 4.8 mm of swelling)} \end{aligned}$$

Now we can assess the lower 25 mm clearance which is affected by the swelling of the joist and the beam:

$$S_{\text{beam}} + S_{\text{joist}} = (12.2 \text{ mm}) + (17.5 \text{ mm}) = 29.7 \text{ mm (swelling)}$$

The swelling of the beam and joist exceeds the 25 mm clearance which means the beam + joist will begin to bear on the bottom of the CLT panel. Ignoring (for now) concerns of bearing strength, the difference beyond the clearance will begin to uplift the CLT. Now we can assess the upper 25 mm clearance:

$$S_{\text{CLT}} + (S_{\text{beam}} + S_{\text{joist}} - 25 \text{ mm}) = (4.8 \text{ mm}) + (29.7 \text{ mm} - 25 \text{ mm}) = 9.5 \text{ mm}$$

Therefore the CLT will uplift but the upper 25 mm clearance will not be exceeded.

#### Concerns with system:

Based on the swelling in the scenario described, a primary concern could be the bearing strength of the CLT, joist, or beam, dependent on some other factors. For example, the joist and beam must be able to withstand the additional bearing forces incurred through uplifting the CLT and its loads. The CLT must also be able to support the change in bearing length from the column support to the width of the joist during

uplift. Alternatively, if the joist or beam is sufficiently weak in bearing, it is possible that the joist begins to crush between the swelling beam and relatively rigid CLT.

## QUESTION 4

Using the O86-14 equation for shrinkage/swelling, we can determine the difference in height between both sides of the top-most beam based on the shrinkage of the undetailed beams.

$$\Delta_{\text{beam, top}} = \Sigma S_{\text{column, I}} = \Sigma [D \times (M_i - M_F) \times C_{\text{parallel}}]$$

$$\Delta_{\text{beam, top}} = [ (4000 \text{ mm}) \times (28\% - 7\%) \times (0.00005) ] \times 3$$

$$\Delta_{\text{beam, top}} = [ (4000 \text{ mm}) \times (28\% - 7\%) \times (0.00005) ] \times 3$$

$$\Delta_{\text{beam, top}} = 12.6 \text{ mm (shrinkage)}$$

Now the induced moment can be determined using the equation for a rigid beam experiencing a vertical displacement at a node:

$$M_{\text{induced}} = 6EI\Delta/L^2 = 6 \times (25600 \times 10^9 \text{ Nmm}^2) \times (12.6\text{mm}) / (9000 \text{ mm})^2$$

$$M_{\text{induced}} = 24 \times 10^6 \text{ Nmm} = 24 \text{ kNm}$$

Therefore the induced moment from shrinkage is about 24 kNm.

## QUESTION 5

### Part A)

In this question we are to determine the effect of water weight on the deflection of the glulam beam. To begin, we can calculate the oven dry (OD) weight of the beam and CLT as a distributed load.

#### Beam:

The OD specific gravity of this grade and species of glulam is given as 0.49. Therefore,

$$\rho_{\text{beam, OD}} = \gamma_{\text{beam, OD}} \rho_{\text{water}}$$

$$\rho_{\text{beam, OD}} = (0.49) \times (1000 \text{ kg/m}^3) = 490 \text{ kg/m}^3$$

Now the weight as a distributed load is:

$$w_{\text{beam, OD}} = \rho_{\text{beam, OD}} b d g = (490 \text{ kg/m}^3) \times (0.215 \text{ m}) \times (0.266 \text{ m}) \times (9.81 \text{ m/s}^2)$$

$$w_{\text{beam, OD}} = 0.275 \text{ kN/m (self-weight of the oven-dry beam)}$$

Moisture content refers to the percentage of water by mass relative to the mass of the wood. Therefore, for 45% saturation, the additional weight of water is 45% the oven dry weight of the beam.

$$w_{\text{beam, wet}} = 1.45 \times (0.275 \text{ kN/m})$$

$$w_{\text{beam, wet}} = 0.40 \text{ kN/m}$$

#### CLT:

The oven dry density of the CLT is given, so we will similarly calculate the oven dry and wet self-weights as distributed loads. The beams are spaced at 2.5 m and the beam in question is an interior beam, so the tributary width of the CLT flooring and corresponding deadload is 2.5 m. The CLT thickness is 7 x 35 mm = 245 mm.

$$w_{\text{CLT, OD}} = \rho_{\text{CLT, OD}} t w_{\text{trib}} = (420 \text{ kg/m}^3) \times (0.245 \text{ m}) \times (2.5 \text{ m}) \times (9.81 \text{ m/s}^2)$$

$$w_{\text{CLT, OD}} = 2.52 \text{ kN/m}$$

Similarly, the wet weight of the CLT at 45% saturation is 45% of the oven dry weight.

$$w_{\text{CLT, wet}} = 1.45 \times (2.52 \text{ kN/m})$$

$$w_{\text{CLT, wet}} = 3.65 \text{ kN/m}$$

#### Deadload:

Similarly, the tributary width for the deadload pressure is 2.5 m based on the glulam beam spacing.

$$w_{\text{dead}} = (32 \text{ kPa}) / (2.5 \text{ m})$$

$$w_{\text{dead}} = 12.8 \text{ kN/m}$$

#### Deflections:

With the wet and dry weights calculated, we can use the equation for midspan deflection of a simply supported beam to calculate the maximum deflections in each case.

Oven Dry:

$$\Delta_{\text{OD}} = 5/384 w_{\text{OD}} L^4/EI$$

$$\Delta_{\text{OD}} = 5/384 \times [ (0.275 \text{ kN/m}) + (2.52 \text{ kN/m}) + (12.8 \text{ kN/m}) ] \times (5 \text{ m})^4 / (4180 \text{ kNm}^2)$$

$$\Delta_{OD} = 30 \text{ mm}$$

45% Moisture:

$$\Delta_{wet} = 5/384 w_{wet} L^4/EI$$

$$\Delta_{wet} = 5/384 \times [ (0.40 \text{ kN/m}) + (3.65 \text{ kN/m}) + (12.8 \text{ kN/m}) ] \times (5 \text{ m})^4 / (4180 \text{ kNm}^2)$$

$$\Delta_{wet} = 33 \text{ mm}$$

Therefore the difference in beam deflection between the OD and 45% moisture case is about 3 mm.

### Part B)

In general, elastic deflection limits are limited to  $L/180$  or, in the case where more than 50% of the load is a long term load,  $L/360$  (see CL 5.4.2 and CL 5.4.3). In this scenario, its interesting to note that even in the oven dry condition, the beam fails the more lenient  $L/180 = 28 \text{ mm}$  for a normal importance factor. Therefore, this system does not adequately meet the serviceability requirements in O86-14.