CWC Wood Engineering
Assignment 2 Solutions

QUESTION 1

Part A) The governing load case will maximize the axial force in member CE. As there only dead and live specified loads, load cases 1 and 2 stand out for comparison. It's important to note that each load case will have a different load duration factor, K_D, per **O86-14 5.3.2**. Therefore, a larger factored load value may not govern a member's design if a lower P_F has $K_D < 1.0$.

Load Case 1: 1.4D

$P_F = 1.4(12\ kN) = 16.8\ kN$
$K_D = 0.65$ (only dead load is applied)

Load Case 2: 1.25D + 1.5 L

$P_F = 1.25(12\ kN) + 1.5(5\ kN) = 22.5\ kN$

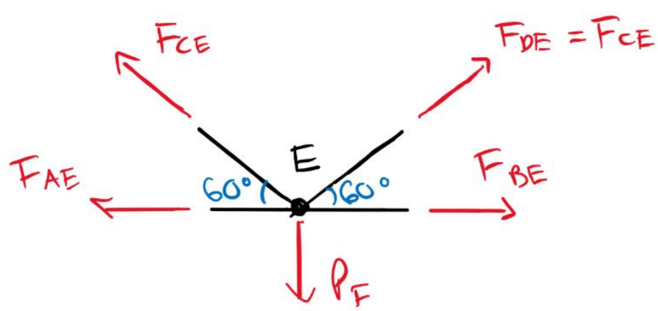
$K_D = 1.0 – 0.5 \log (P_L / P_S) \geq 0.65$
$K_D = 1.0 – 0.5 \log (12\ kN/5\ kN) \geq 0.65$
$K_D = 0.81$

When the specific dead (permanent) load exceeds the specified (unfactored) standard term loads (snow and live in the combinations shown in **5.3.2.3**), K_D is calculated as follows. Note that only the loads considered in the relevant load case you are using need to be included.

As mentioned, for different K_D values, the governing load case may not be obvious since K_D reduces the capacity of a member. One solution for this is to estimate the "apparent" factored load by dividing P_F/K_D which allows for a comparison between load cases with different K_D values (this will not work for members in axial compression since K_D is non-linearly related to the factored resistance). Therefore,

Load Case 1: $P_F/K_D = 16.8\ kN/0.65 = 25.8\ kN$
Load Case 2: $P_F/K_D = 22.5\ kN/0.81 =$ **27.8 kN (governs)**

Part B) First the tension in member CE must be determined. Since the system is symmetric, the forces in CE and DE will be the same. Examining equilibrium at Node E:



$\Sigma F_Y = 0 = -P_F + F_{CE}\sin 60 + F_{DE}\sin 60$
$F_{CE} = P_F/(2\sin 60)$
$F_{CE} = 22.5 \text{ kN} / (2\sin 60)$
$F_{CE} = 13.0 \text{ kN}$ (tension)

Now we can use the selection tables to select a 38 mm Nothern No. 1 member for the factored load in CE. Similarly to above, its important to note that the Wood Design Manual selection tables offer resistance values for standard conditions only (typically meaning standard load durations, dry service conditions, no treatments, basic loading configurations, and etc.). Therefore, any strength reducing/increasing factors must be accounted for manually.

For the purposes of selecting a member from the tables, we can adjust the force calculated for member CE by the load duration factor for our governing case as well as the assumed reduction in gross area of 15% for connections (both factors reduce the capacity of a member and will increase the "apparent" load).

For use in the member selection tables:

$T_{R,\text{ selection}} \geq F_{CE}/K_D(A_N/A_G) = (13.0 \text{ kN})/[(0.81)(0.85)]$
$T_{R,\text{ selection}} \geq 18.9 \text{ kN}$

From the selection tables for 38 mm Nothern No. 1 Sawn Timber Tension members, the minimum suitable size appears to be 38x140 with $T_R = 24.9$ kN.

Design Check for 38 x 140 mm:
Per **Table 6.3.1.A**, member is "Structural Joist + Plank".

Member Properties (**Table 6.4.5**):

$f_t = 4.0$ MPa (tensile strength)

Calculating Sawn Lumber Tensile Resistance Parallel to Grain (**6.5.9**)

$T_R = \phi F_t A_n K_{zt} \geq F_{CE}$

$F_t = f_t\,(K_D K_H K_{st} K_T)$
$F_t = (4.0 \text{ MPa})(0.81)(1)(1)(1)$
$F_t = 3.24$ MPa

$K_{zt} = 1.3$ (**Table 6.4.5**)

$T_R = (0.9)(3.24 \text{ MPa})(0.85 \times 38 \times 140 \text{ mm}^2)(1.3) \geq 13.0 \text{ kN}$
$T_R = 17.1 \text{ kN} \geq 13.0 \text{ kN}$ (**Passes**)

This section passes the tensile design check, but is it the smallest possible member per the original question? Check one size lower: 38 x 89 mm

Design Check for 38 x 89 mm:

$K_{zt} = 1.5$ (**Table 6.4.5**)

T_R = (0.9)(3.24 MPa)(0.85 x 38 x 89 mm^2)(1.5) ≥ 13.0 kN
T_R = 12.6 kN ≥ 13.0 kN (**FAILS**)

The new section fails the tension design check. Therefore, 38 x 140 mm is the smallest suitable member of this grade and species.

QUESTION 2

Here, we are to select a column depth for a 16c-E D. Fir-L member with the bracing and loading shown and in wet conditions. This will prove more challenging than question 1 as we cannot easily compensate for varying K_D values for axial compression members.

Considering the following load combinations, we get:

Load Case 1:

$P_{f1} = 1.4D = 1.4(300\ kN) = 420\ kN$
$K_D = 0.65$

Load Case 2:

$P_{f2} = 1.25D + 1.5L + 1.0S = 1.25(300\ kN) + 1.5(100\ kN) + 1.0(150kN) = 675\ kN$

The greatest standard term load P_s is calculated per **5.3.2.3**.

$P_S = S + 0.5L = 200\ kN$

$K_D = 1.0 – 0.5\log(P_L/P_S) = 1.0 – 0.5\log(300/200)$
$K_D = 0.91$

Load Case 3:

$P_{f3} = 1.25D + 1.5S + 1.0S = 1.25(300\ kN) + 1.5(150\ kN) + 1.0(100\ kN) = 700\ kN$

For the same standard and long-term loads as Case 2, K_D will be the same.

$K_D = 0.91$

For the same K_D, we can distinguish that Load Case 3 will govern over Load Case 2. We must proceed with full design calculations to determine which will govern between Case 1 and 3.

Design Check:
The column is in wet service conditions which yields the following reduction factors from **Table 7.4.2.**

$K_{SC} = 0.75$; $K_{SE} = 0.9$

Based on these reduction factors, the compression resistance will be reduced by at least 0.75 compared to the tabulated values. We can take an educated guess and conservatively choose a trial member from the selection tables. Try 215x304 mm 16c-E D. Fir-L (Table P_r = 902 to 1320 kN for L = 2.0 to 4.0 m)

Member Properties (**Table 7.3**):

$f_C = 30.2$ MPa
E = 12.4 GPa

For glulam columns (not sawn lumber), the compression design is governed by the greatest axial slenderness ratio calculated in **7.5.8**. Only the most slender axis need be considered.

Strong Axis:

$C_{CX} = L_X K_e / d$

Note that the product LK_e is the effective column length and relies on the shortest unbraced length of the column. Here, there is intermediate support in both axes, and $K_e = 1.0$ for pin-pin supports (**A.6.5.6.1**).

$C_{CX} = (4.0\text{ m})(1.0)/(0.304\text{ m}) = 13.3 \leq 50$

Weak Axis:

$C_{CY} = L_Y K_e / b$

$C_{CY} = (3.0\text{ m})(1.0) / (0.215\text{ m}) = 14.0 \leq 50$ (Weak axis governs design)

For axial compression resistance parallel to grain:

$P_R = \phi F_C A K_{zcg} K_C$

$K_{zcg} = 0.68(\text{beam volume})^{-0.13} \leq 1.0$

$K_{zcg} = 0.68(8000 \times 304 \times 215\text{ mm}^3)^{-0.13} \leq 1.0$

$K_{zcg} = 0.74 \leq 1.0$

Now for F_C and K_C the calculations will diverge depending on our potentially governing load cases:

Load Case 1:

$F_C = f_c K_D K_H K_{SC} K_T = (30.2\text{ MPa})(0.65)(1.0)(0.75)(1.0) = 14.7\text{ MPa}$

$K_C = [1.0 + F_C K_{zcg} C_C^3 / 35 E_{05} K_{se} K_T]^{-1}$

$E_{05} = 0.87E = 0.87(12.4\text{ GPa})$

$K_C = [1.0 + (14.7\text{ MPa})(0.74)(14.0)^3/35(10788\text{ MPa})(0.90)(1.0)]^{-1}$

$K_C = 0.92$

Now:

$P_{r1} = (0.80)(14.7\text{ MPa})(215 \times 304\text{ mm}^2)(0.74)(0.92) \geq P_{f1}$

$P_{r1} = 523\text{ kN} \geq 420\text{ kN}$ (**Passes**)

This member is suitable for Load Case 1.

Load Case 3:

$F_C = (30.2\text{ MPa})(0.91)(1.0)(0.75)(1.0) = 20.6\text{ MPa}$

$K_C = [1.0 + (20.6\text{ MPa})(0.74)(14.0)^3 / 35(10788\text{ MPa})(0.90)(1.0)]^{-1}$

$K_C = 0.89$

Now:

$P_{r3} = (0.80)(20.6\text{ MPa})(215 \times 304\text{ mm}^2)(0.74)(0.89) \geq P_{f3}$

$P_{r3} = 709\text{ kN} \geq 700\text{ kN}$ (**Passes**)

This member is suitable for Load Case 3.

This section passes the compression design check for both load cases. It is interesting to observe the ~200 kN difference in compressive strength resulting from varying K_D factors. This column is reasonably efficient with a utilization of 700/709 = 0.99.

QUESTION 3

Key Assumptions:

-Wet service conditions

-All joist loads are the same

-Only joist bearing may govern (shear and flexure assumed adequate). The joists rest on half of the girder's width

-Only girder bearing, shear, moment, or deflection may govern (connections and columns are adequate)

-$P_L = 0.5P_D$

Therefore, the bearing capacity of the joists and the moment, shear, deflection, and bearing capacity of the girder must be calculated to determine the critical P_f.

For dead and live loads only, the following load cases may govern. Since axial compression is not considered, we can determine the governing load case based on K_D.

<u>Load Case 1:</u>

$P_f = 1.4D = 1.4P_D$

$K_D = 0.65$

$P_f/K_D = 2.15P_D$

<u>Load Case 2:</u>

$P_f = 1.25D + 1.5L = 1.25P_D + 1.5(0.5P_D) = 2P_D$

$K_D = 1.0 – 0.5\log(P_D/0.5P_D) = 0.85$

$P_f/K_D = 2.35P_D$ (**governs over load case 1**)

Therefore, Load Case 2 will govern.

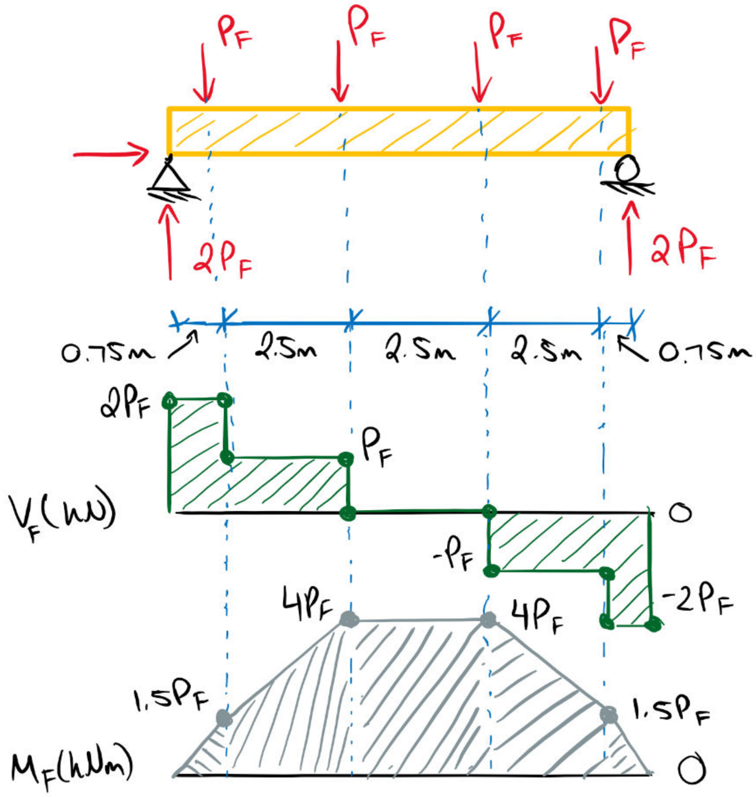
Girder Properties: 215x950 SPF 20f-E

$f_b = 25.6$ MPa; $f_v = 1.75$ MPa; $f_{cp} = 5.8$ MPa; $f_{tb} = 5.8$ MPa; E = 10300 MPa
$K_{SV} = 0.87$; $K_{SE} = 0.90$; $K_{Sb} = 0.80$; $K_{Scp} = 0.67$

Joist Properties: 191x343 D. Fir-L Select Structural

$f_{cp} = 7.0$ MPa
$K_{Scp} = 0.67$

Shear/Moment Diagram:



$M_{f,\,max} = 4P_f$
$V_{f,\,max} = P_f$ for a distance d away from the support per **7.5.7.1.1**

Girder Shear Resistance (**7.5.7.2**):

The beam volume must be checked to determine the appropriate shear calculation per **7.5.7.2**

$Z = bdl = 0.215 \times 0.950 \times 9\ m^3 = 1.84\ m^3 < 2.0\ m^3$

The volume is less than 2.0 m^3 so we can use the simplified shear calculation in **7.5.7.2.b**

$V_r = \phi F_V A_G 2/3$

$F_v = f_v K_D K_H K_{sv} K_T = (1.75\ MPa)(0.85)(1.0)(0.87)(1.0) = 1.29\ MPa$

$V_r = (0.90)(1.29\ MPa)(215 \times 950\ mm^2)2/3 \geq V_f = P_f$
$V_r = 158\ kN \geq P_f$

Girder Moment Resistance (**7.5.6.5**):
This beam only has one point of inflection in the factored applied moment. Therefore, no special considerations are needed per the note in **7.5.6.5**.

Two moment resistance calculations are offered in O86-14, M_{r1}, which checks the moment resistance in pure flexure with a failure mode in extreme fibre failure, and M_{r2}, which checks the moment resistance for failure in lateral torsional buckling. This is why M_{r2} utilizes the lateral stability factor K_L.

$M_{r1} = \phi F_b S K_x K_{zbg}$
$M_{r2} = \phi F_b S K_x K_L$

$F_b = f_b K_D K_H K_{sb} K_T$ = (25.6 MPa)(0.85)(1.0)(0.80)(1.0) = 17.4 MPa

$S = bd^2/6$ = (215 mm)(950 mm)^2/6 = 32.34 x 10^6 mm^3

The size factor for glulam in bending applies the width of the widest laminate (taken as b/2 = 107.5 mm in this example).

$K_{zbg} = [130/b \times 610/d \times 9100/L]^{1/10} \leq 1.3$
K_{zbg} = [130/(107.5 mm) x 610/(950 mm) x 9100/(9000 mm)]$^{1/10}$ ≤ 1.3
$K_{zbg} = 0.98 \leq 1.3$

For the lateral stability factor we can assume the joist loads stabilize the member which makes the joist spacing the maximum unsupported beam length (**7.5.6.4.3**). Therefore,

a = 2.5 m

We can calculate the effective beam length L_e from **Table 7.5.6.4.3**. The loading condition in this example doesn't fit into a category for evenly spaced point loads (maybe it falls between 1/3 to 1/5 points – judgement call) so we will be conservative and use the calculation for "any loading".

L_e = 1.92a = 1.92(2.5 m) = 4.8 m

The slenderness ratio is:

$C_B = [L_e d/b^2]^{1/2}$ = [(4.8 m)(0.95 m)/(0.215 m)^2]$^{1/2}$ = 9.93 ≤ 50

Now from **7.5.6.4.4** for $C_B < 10$:

K_L = 1.0 (our conservative L_e calculation likely would not have change much).

Now we can calculate the M_r.

M_{r1} = (0.90)(17.4 MPa)(32.34x10^6 mm^3)(1.0)(0.98)
M_{r1} = 496 kNm (**governs M_r**)

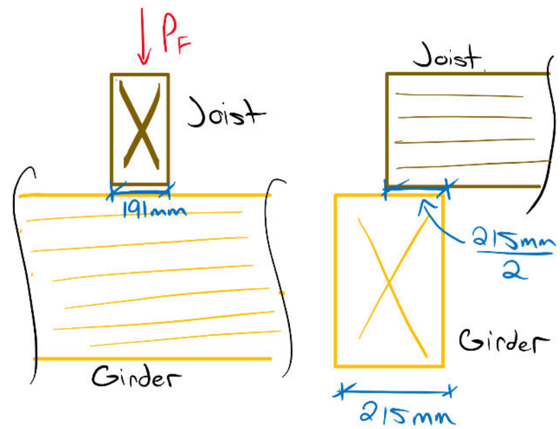
M_{r2} = (0.90)(17.4 MPa)(32.34x10^6 mm^3)(1.0)(1.0)
M_{r2} = 506 kNm

Therefore,

M_r = 496 kNm ≥ $M_f = 4P_f$

Girder Bearing (**7.5.9**):
The bearing on the girder must be considered in two capacities. The first, governed by **7.5.9.2** is the bearing at the applied loads from the joists away from the support. The second, governed by **7.5.9.3** is the bearing within a distance d from the center of the support (also called critical bearing). In this region, the beam is "squeezed" between the applied load and the support which causes a different bearing mechanism than the first case.

Bearing at the Joist Load (away from the support – **7.5.9.2**):



$Q_r = \phi F_{cp} A_b K_B K_{zcp}$

$F_{cp} = f_{cp} K_D K_{scp} K_T = (5.8 \text{ MPa})(0.85)(0.67)(1.0) = 3.3 \text{ MPa}$

$A_B = 191 \text{ mm} \times 215 \text{ mm}/2 = 20532.5 \text{ mm}^2$

The size factor for bearing is determined in **6.5.7.4** using the laminate thickness as the depth.

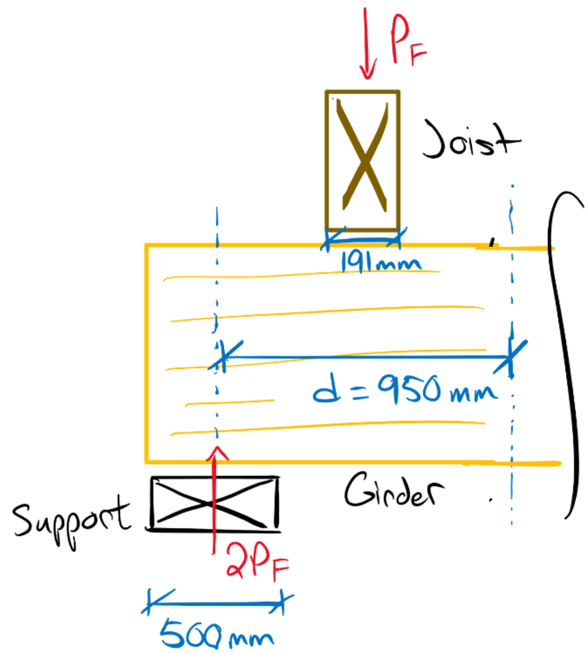
For $b/d = (215 \text{ mm})/(38 \text{ mm}) = 5.7 \geq 2.0$,
$K_{zcp} = 1.15$

The length of bearing factor K_B is determined by **6.5.7.5**. Since the middle two joists are in the region of highest bending stress, the benefit of locally stressing fibres to resist bearing should be ignored.

$K_B = 1.0$

$Q_{r,\text{ load}} = (0.80)(3.3 \text{ MPa})(20532.5 \text{ mm}^2)(1.0)(1.15) \geq Q_f$
$Q_{r,\text{ load}} = 62 \text{ kN} \geq Q_f = P_f$

Bearing at the Support Reaction (**7.5.9.2**):



A_b = 500 mm x 215 mm = 107500 mm^2

Since the bearing length falls within 75 mm from the end of the member, no strength bonus from the length of bearing is permitted.

K_B = 1.0

$Q_{r,\, support}$ = (0.80)(3.3 MPa)(107500 mm^2)(1.0)(1.15) ≥ Q_f
$Q_{r,\, support}$ = 326 kN≥ $Q_f = 2P_f$

Critical Bearing at the Joist Load (near support – **7.5.9.3**):

$Q_r' = 2/3\ \phi F_{cp} A_b' K_B K_{zcp}$

Since the bearing area is away from the member end and not in a region of high bending stress, we can get K_B from **Table 6.5.7.5**. However, for a bearing length greater than 150 mm (191 mm), no increase is permitted.

K_B = 1.0

The average bearing area between the support and the load near the support is needed (**6.5.7.3.2**):

$A_b' = b\ (L_{b1} + L_{b2})/2 \leq 1.5bL_{b1}$

b = (215 mm + 107.5 mm)/2 = 161.25 mm

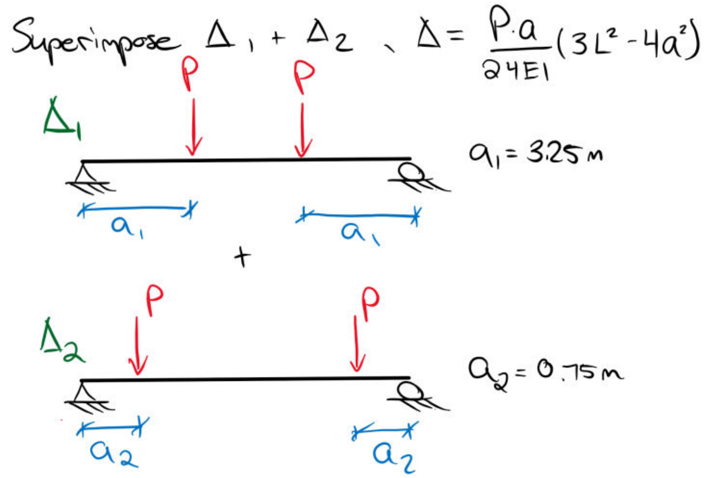
A_b' = (161.25 mm) [(191 mm)(500 mm)]/2 ≤ 1.5(161.25 mm)(191 mm)
A_b' = 55712 mm^2 ≤ 46198 mm^2
A_b' = 46198 mm^2

$Q_r' = 2/3\ (0.80)(3.3\ \text{MPa})(46198\ \text{mm}^2)(1.0)(1.15) \geq Q_f$
$Q_r' = 93.5\ \text{kN} \geq Q_f = P_f$

Girder Deflection:
The deflection limit of L/360 may also govern the factored load P_f. Per **5.4.1**, the modulus of elasticity needs to be modified by K_{SE} for deflection calculations. For the deflections of a beam in the loading conditions shown, we can rely on the Beam Diagrams in the reference information section of the CWC Wood Design Manual. While there is no diagram for the loading specific in this example, for elastic deflections we can use the superposition of two states to calculate the total deflection.

$E_s = EK_{SE}K_T = (10300\ \text{MPa})(0.90)(1.0) = 9270\ \text{MPa}$

$E_sI = (9270\ \text{MPa})(215\ \text{mm})(950\ \text{mm})^3/12 = 142400\ \text{kNm}^2$



$\Delta_{total} = \Delta_1 + \Delta_2 \leq L/360$
$\Delta_{total} = Pa_1\ (3L^2-4a_1^2)/24EI + Pa_2\ (3L^2-4a_2^2)/24EI$
$P \leq 24E_sIL/360 \times [a_1\ (3L^2-4a_1^2)+ a_2\ (3L^2-4a_2^2)]^{-1}$
$P \leq 24(142400\ \text{kNm}^2)(9\ \text{m})/360 \times [\ (3.25\ \text{m})(3(9\ \text{m})^2 - 4(3.25\ \text{m})^2) + (0.75\ \text{m})(3(9\ \text{m})^2 - 4(0.75\ \text{m})^2)\ ]^{-1}$
$P \leq 102\ \text{kN}$

Now we must finally check the bearing of the joist member.

Joist Bearing (**6.5.7**):
Since there are no bearing loads applied within a distance d from the support (the joist supports a distributed load), the only bearing check needed is for the support. Therefore, 6.5.7.2 applies. The length of bearing and size factor must be calculated for this scenario.

$Q_{r,\ joist} = \phi F_{cp}A_bK_BK_{zcp}$

For $b/d = 191/343 < 1.0$,
$K_{zcp} = 1.0$

Per **6.5.7.5**, since the bearing area occurs at the end of the member, no strength benefit can be claimed.

$K_B = 1.0$

$F_{cp} = f_{cp}K_DK_{sp}K_T$ = (7.0 MPa)(0.85)(0.67)(1.0) = 4.0 MPa

$Q_{r,\ joist}$ = (0.80)(4.0 MPa)(107.5x191 mm^2)(1.0)(1.0) ≥ Q_f
$Q_{r,\ joist}$ = 65.5 kN ≥ Q_f = P_f

Summary of Governing Loads:

Based on the various resistances and limits calculated, we can determine the governing P_D, P_L using the relationships determined earlier $P_f = 2P_D$, $P_L = 0.5P_L$, and $P = 1.5P_D$.

Girder Shear:

V_r = 158 kN ≥ P_f
P_f ≤ 158 kN
P_D = 79 kN, P_L = 39.5 kN

Girder Moment:

M_r = 496 kNm ≥ $4P_f$
P_f ≤ 124 kN
P_D = 62 kN, P_L = 31 kN

Girder Deflection:

Δ_{total} ≤ L/360
P ≤ 102 kN = $1.5P_D$
P_D = 68 kN , P_L = 34 kN

Girder Bearing @ Joist Load away from support:

$Q_{r,\ load}$ = 62 kN ≥ P_f
P_f ≤ 62 kN
P_D = 31 kN , P_L = 15.5 kN (**Governs**)

Girder Bearing @ Support Reaction:

$Q_{r,\ support}$ = 326 kN≥ $2P_f$
P_f ≤ 163 kN
P_D = 81.5 kN, P_L = 40.8 kN

Girder Critical Bearing Near Support:

Q_r' = 93.5 kN ≥ P_f
P_f ≤ 93.5 kN
P_D = 46.8 kN, P_L = 23.4 kN

Therefore, based on this system, the bearing at a joist load away from the support is the governing case and limits P_D to 31 kN and P_L to 15.5 kN.

QUESTION 4

This configuration introduces bearing at an angle to the grain direction of the 241x241 member. Therefore, **6.5.8** applies which requires a combination of both bearing (perpendicular to grain) and compression (parallel to grain) resistances for the member with some basic modifications.

$f_c = 8.7$ MPa
$f_{cp} = 5.3$ MPa
$K_{zcp} = 1.0$ (assumed in question)

For bearing at an angle to the grain:

$N_r = P_r Q_r / (P_r \sin^2\theta + Q_r \cos^2 \theta)$

where P_r is calculated assuming $K_C = 1.0$

<u>Resistance Parallel to grain (**6.5.6.2.3**):</u>

$F_c = f_c K_D K_H K_{sc} K_T = (8.7 \text{ MPa})(1.0)(1.0)(1.0)(1.0) = 8.7$ MPa

$P_r = \phi F_c A K_{zc} K_C = (0.80)(8.7 \text{ MPa})(241\text{x}241 \text{ mm}^2)(1.0)(1.0)$
$\mathbf{P_r = 404}$ **kN**

<u>Resistance Perpendicular to grain (**6.5.7.2**):</u>

$F_{cp} = f_{cp} K_D K_{scp} K_T = (5.3 \text{ MPa})(1.0)(1.0)(1.0) = 5.3$ MPa

$Q_r = \phi F_{cp} A_b K_B K_{zcp}$

The bearing length parallel to grain is:

$L_b = 191 \text{ mm}/\sin 30° = 382 \text{ mm} > 150 \text{ mm}$

$K_B = 1.0$

For $b/d = 241/241 \leq 1.0$,
$K_{zcp} = 1.0$

$Q_r = (0.80)(5.3 \text{ MPa})(382\text{x}141 \text{ mm}^2)(1.0)(1.0)$
$\mathbf{Q_r = 228}$ **kN**

<u>Total bearing resistance at an angle to grain:</u>

$N_r = (404 \text{ kN})(228 \text{ kN})/[\ (404 \text{ kN})\sin^2(30) + (228 \text{ kN})\cos^2(30)\ ]$
$N_r = 339$ kN

Therefore the bearing resistance of the beam for this configuration is 339 kN.

QUESTION 5

In this example, we are to determine the minimum number of plies needed for the built-up sawn cantilever column assuming the nail requirements are met per **6.5.6.4.2**. We can use two properties of built-up columns to solve for the number of plies: 1) **6.5.6.4.6** allows the strong axis to see no reduction in compressive strength and 2) **6.5.6.4.2** allows 60% of the gross section compressive strength for the weak axis to be taken. Therefore, we can solve for the governing weak axis member depth using these relationships.

Member Properties: 38x184 D.Fir-L No. 2

f_c = 14.0 MPa; E_{05} = 7000 MPa

K_{TE} = 0.95; K_T = 0.85 (Table 6.4.3)

K_{sc} = 0.69; K_{SE} = 0.94 (Table 6.4.2)

K_D = 1.0

<u>Strong Axis Compression:</u>
The strong axis compressive resistance is taken as the full resistance of the gross section. For a cantilever column, the K_e can be found in **A.6.5.6.1**.

$C_{cx} = K_eL/d = (2.0)(3.5\text{ m})/(0.184\text{ mm}) = 38.0 \leq 50$

$K_{zc} = 6.3(dL)^{-0.13} \leq 1.3$
$K_{zc} = 6.3(184\text{ mm x }3500\text{ mm})^{-0.13} \leq 1.3$
$K_{zc} = 1.1 \leq 1.3$

$F_c = f_cK_DK_HK_{sc}K_T = (14.0\text{ MPa})(1.0)(1.0)(0.69)(0.85) = 8.21\text{ MPa}$

$K_c = [1.0 + F_CK_{zcg}C_C^3 / 35E_{05}K_{se}K_T\ ]^{-1}$
$K_c = [1.0 + (8.21\text{ MPa})(1.1)(38.0)^3 / 35(7000\text{ MPa})(0.94)(0.85)]^1$
$K_c = 0.283$

$P_{rx} = \phi F_cAK_{zc}K_c \text{ x } n \geq P_f = 20\text{ kN}$
$n \geq (20000\text{ N})/(0.8)(8.21\text{ MPa})(38\text{x}184\text{ mm}^2)(1.1)(0.283)$
$n \geq 1.4$ plies
n = 2 plies for sufficient strong axis P_r

<u>Weak Axis:</u>
Weak axis P_r is taken as 60% of the built-up gross section.

$C_{cy} = K_eL/nb = (2.0)(3.5\text{ m})/n(0.038\text{ m}) \leq 50$
$n \geq (2.0)(3.5\text{ m})/50(0.038\text{ m})$
$n \geq 3.68$ plies
n = 4 plies to meet slenderness requirements

Proceed resistance check for 4 plies:

C_{cy} = (2.0)(3.5 m)/(4)(0.038 m) = 46 ≤ 50

$K_c = [1.0 + (8.21\ MPa)(1.1)(46.0)^3 / 35(7000\ MPa)(0.94)(0.85)]^1$
K_c = 0.182

$P_{ry} = 0.60 \times \phi F_c A K_{zc} K_c \times n \geq P_f = 20\ kN$
$P_{ry} = 0.60 \times (0.80)(8.21\ MPa)(38\text{x}184\ mm^2)(1.1)(0.182) \times 4\ plies \geq P_f = 20\ kN$
$P_{ry} = 22\ kN \geq P_f = 20\ kN$

Therefore, 4 plies is the minimum number to resist P_f for a fixed member size of 38x184.

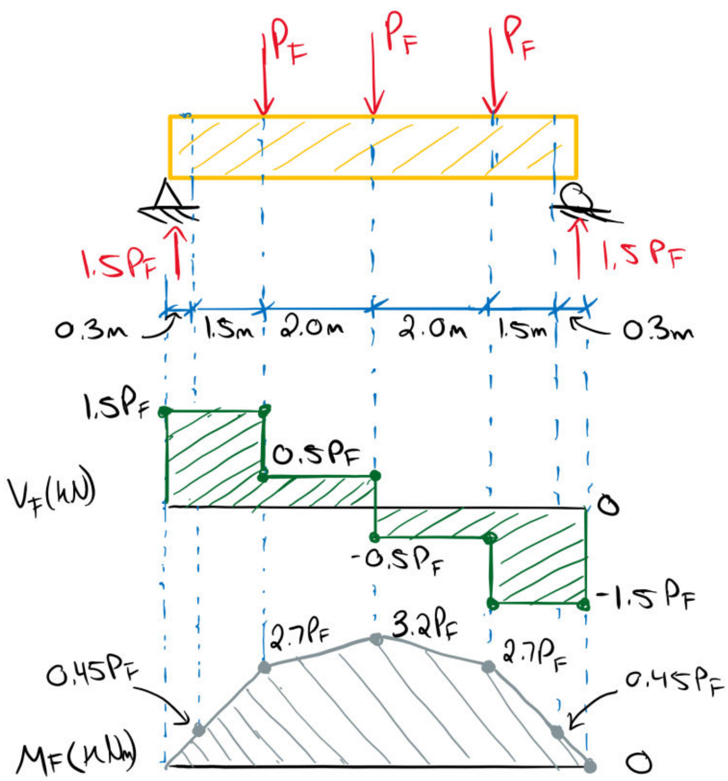
QUESTION 6

This question involves determining the governing beam resistance in both cases when deflections and bearing are not considered. Therefore, the possible governing cases are:

- Beam Flexure (same resistance in both cases)
- Beam Shear (same resistance in both cases)
- Tension Notch Shear (Case A)
- Compression Notch Shear (Case B)

Beam Properties: 365x798 D.Fir-L 24f-E

Laminates are b/2 x 38 mm
f_b = 30.6 MPa; f_v = 2.0 MPa; E = 12800 MPa
No treatment, dry service conditions
K_D = 1.15



Moment Resistance:

The moment resistance will be checked for both the midspan and the reduced cross-section as a precaution.

Full Depth:

$S = bd^2/6 = 38.73 \times 10^6$ mm^3

$F_b = f_bK_DK_HK_{sb}K_T$ = (30.6 MPa(1.15)(1.0)(1.0)(1.0) = 35.2 MPa

K_{zbg} = [130/b x 610/d x 9100/L]$^{1/10}$ ≤ 1.3
K_{zbg} = [130/(182.5 mm) x 610/(798 mm) x 9100/(7600mm)]$^{1/10}$ ≤ 1.3
K_{zbg} = 0.96 ≤ 1.3

Assuming the purlins provide lateral stability, the maximum purlin spacing is 2.0 m. Due to irregular load spacing we will conservatively apply the "any loading" condition in **Table 7.5.6.4.3**.

a = 2.0 m
L_e = 1.92a = 3.84 m

$C_B = [L_e d/b^2]^{1/2}$ = [(3.84 m)(0.798 m)/(0.365 m)^2]$^{1/2}$ = 4.8 ≤ 50

For C_B ≤ 10 in **7.5.6.4.4**

K_L = 1.0

$M_{r1} = \phi F_b S K_{zbg} K_x$
M_{r1} = (0.9)(35.2 MPa)(38.73 x 10^6 mm^3)(0.96)(1.0)
M_{r1} = 1179 kNm ≥ M_f = 3.2P_f (governs M_r)

$M_{r2} = \phi F_b S K_L K_x$
M_{r2} = (0.9)(35.2 MPa)(38.73 x 10^6 mm^3)(1.0)(1.0)
M_{r2} = 1227 kNm ≥ M_f = 3.2P_f

After Notch:
As an exercise, the moment resistance at the notched section can be compared to the factored moment in the same location. Calculating the moment resistance after the notch requires some discretion since the relevant M_r calculations consider lateral torsional buckling and extreme fibre failures. For a small segment of the beam, it may not be appropriate to recalculate K_L and K_{zbg} in the sense that the risk of lateral torsional buckling over a small section near a support may not be increased sufficiently to warrant determining a new C_B for the reduced section, nor a new size factor. As an estimate, we can adjust the governing M_r for the full beam depth through adjusting the section modulus for the reduced beam depth.

$S_{reduced}$ = (365 mm)(684 mm)^2/6 = 28.46 x 10^6 mm^3
$M_{r1, notch}$ = (0.9)(35.2 MPa)(28.46 x 10^6 mm^3)(0.96)(1.0)
$M_{r1, notch}$ = 866 kN ≥ M_f = 0.45P_f

Therefore, the notching resulted in 66/1179 = 0.73 of the moment resistance but is compensated by 0.45P_f/3.2P_f = 0.14 of the factored moment. Therefore, the drop in factored moment at the notch is more significant than the loss of moment resistance from the notch.

Longitudinal Shear Resistance (**7.5.7**):
The longitudinal shear resistance will be the same for both configurations. The volume of the beam must be checked to determine if the simplified shear procedure is valid.

Z = Lbd = (7.8 m)(0.365 m)(0.798 m) = 2.3 m^3 ≥ 2.0 m^3

The beam volume is greater than 2.0 m^3 therefore the longitudinal shear resistance is governed by **7.5.7.2a**.

$W_r = \phi F_v 0.48 A_g C_v Z^{-0.18} \geq W_f$

Where W_f is all factored loads acting perpendicular to the member, not just the maximum V_f as done previously for the simplified shear procedure.

$F_v = f_v K_D K_H K_{sv} K_T = (2.0\text{ MPa})(1.15)(1.0)(1.0)(1.0) = 2.3\text{ MPa}$

The shear load coefficient C_v is determined in **7.5.7.5**. Since the loading in this example is not exactly specified by **Table 7.5.7.5A**, the procedure in **7.5.7.5a-d** must be followed.

a) See V_f diagram above

b) There are a total of 4 segments each representing a constant value of V_f such that there is no abrupt change in the factored shear.

c) Calculate the factor G for each segment

Segment 1:

$V_A = 1.5P_f$; $V_B = 1.5P_f$; $V_C = 1.5P_f$; $l_a = 1.8$ m

$G_1 = l_a[V_A^5 + V_B^5 + 4V_C^5] = 82P_f^5$ mkN^5

Segment 2:

$V_A = V_B = V_C = 0.5P_f$; $l_a = 2.0$ m

$G_2 = l_a[V_A^5 + V_B^5 + 4V_C^5] = 0.375P_f^5$

Segment 3:

Same as segment 2

$G_3 = 82P_f^5$

Segment 4:

Same as segment 1

$G_4 = 0.375P_f^5$

d) Calculate C_v. Pay attention to the units to assure yourself the final C_v value is unitless. The variable load P_f cancels out of the equation.

$C_v = 1.825W_f\,(L/\Sigma G)^{1/5}$

$W_f = 3P_f$ (sum of all normal loads)

$C_v = 1.825(3P_f)[\,(7.6\text{ m})/(82P_f^5 + 0.375P_f^5 + 0.375P_f^5 + 82Pf^5)\text{mkN}^5\,]^{1/5}$

$C_v = 2.96$

As an exercise, let's assume the loading in this example is symmetrically placed such that we can determine C_v with **Table 7.5.7.5A**. Here we must calculate the ratio of concentrated loads to uniform loads, r*.

$r^* = 3P_f / 0$

As there are no uniform loads, the ratio is undefined and becomes infinity which falls under the "10.0 and over" category. Then, for three concentrated loads.

$C_v = 2.84$

Therefore we achieve a slight increase in C_v (and therefore shear resistance) based on this specific non-standard loading condition.

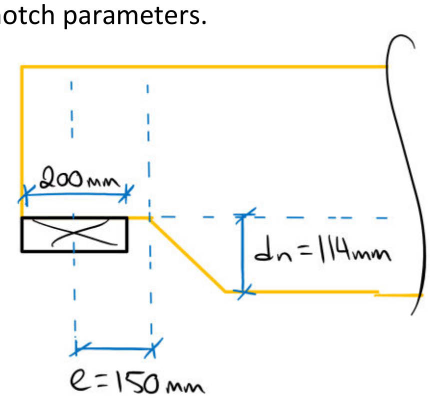
$W_r = (0.90)(2.3\ MPa)0.48(365 \times 798\ mm^2)(2.96)(2.3\ m^3)^{-0.18} \geq W_f = 3P_f$

$\mathbf{W_r = 737\ kN \geq 3P_f}$

Now the notch shear resistance in both cases must be calculated to assess the critical factored load.

Case a) Tension Side Notch Shear:

For tension-side notches, the notch shear resistance is governed by **7.5.7.4**. Per **7.5.7.4.1**, notches with depths less than a quarter of the beam's depth are permitted within a distance equal to the beam's depth with no reduction in the longitudinal shear resistance calculated in **7.5.7.2**. **Figure 6.5.5.3.2** provides some help in determining tension notch parameters.



The fracture shear resistance at a tension-side notch is calculated as follows:

$F_r = \phi F_f A_g K_N$

$F_f = f_f K_D K_H K_{sf} K_T$

$f_f = \max(2.5 b_{eff}^{-0.2}$ or $0.9\ MPa)$

b_{eff} is taken as the width of the widest lamination (in this case b/2 = 182.5 mm)

$f_r = \max(\ 2.5(182.5\ mm)^{-0.2}\ ,\ 0.9\ MPa) = \max\ (0.88\ MPa\ ,\ 0.9\ MPa) = 0.9\ MPa$

$F_f = (0.9\ MPa)(1.15)(1.0)(1.0)(1.0) = 1.035\ MPa$

The Notch Factor, K_N, can be determined through the equation given in **7.5.7.4.2** (the same as **6.5.5.3.2**) or alternatively through interpolation in **Table 6.5.5.3.2** with the notch dimensional parameters.

$K_N = [\ 0.006d\ (\ 1.6\ (1/\alpha - 1) + \eta^2\ (1/\ \alpha^3 - 1))]^{-1/2}$

$\alpha = 1 - d_n/d = 1 - (114\ mm)/(798\ mm) = 0.857$

$\eta = e/d = (150\ mm)/(798\ mm) = 0.188$

$d_n = 114$ mm ; $e = 150$ mm (see **Figure 6.5.5.3.2**)

$K_N = [\ 0.006(798\ mm)\ (\ 1.6\ (1/(0.857) - 1) + (0.188)^2\ (1/\ (0.857)^3 - 1))]^{-1/2}$

$K_N = 0.85$

As an exercise, we can determine K_N through **Table 6.5.5.3.2** with interpolation. Both methods are valid however the interpolation method may prove faster if the specific values of α and η are close to the reported values. Otherwise single or double interpolation may be required. In this example, double interpolation is required for a K_N corresponding to $\alpha = 0.857$, $\eta = 0.188$.

Interpolating for $K_N d^{1/2}(\alpha = 0.857, \eta = 0.15) = 24.6$
Interpolating for $K_N d^{1/2}(\alpha = 0.857, \eta = 0.20) = 24.1$
Now, Interpolating between these values for $K_N d^{1/2}(\alpha = 0.857, \eta = 0.188) = 24.2$

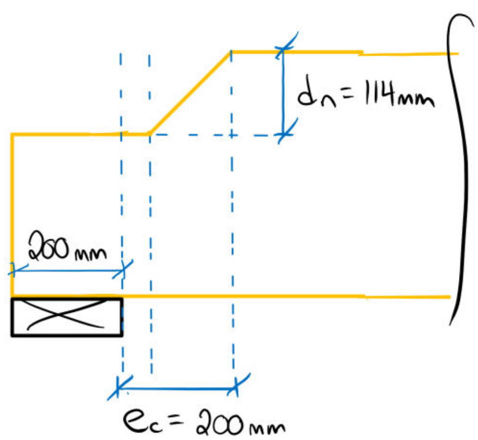
Therefore,

$K_N d^{1/2} = 24.2$
$K_N = 24.2/(798 \text{ mm})^{1/2} = 0.86$

This value can be compared to that determine with the equation in **7.5.7.4.2**. Note the values are very similar and either method can be used to determine K_N.

$F_r = (0.90)(1.035 \text{ MPa})(365 \times 798 \text{ mm}^2)(0.85)$
$\mathbf{F_r = 231\ kN \geq 1.5P_f}$

Case b) Compression Side Notch Shear:
Notch shear resistance for compression-side notches is determined by **7.5.7.3**. For compression notches, note the value of e_c is different than the value of e for tension-side notches, as e_c is measured from the edge of the support face to the end of the notch and e is taken from the center of the support to the beginning of the notch.



$e_c = 200$ mm; $d_n = 114$ mm

For $e_c = 200 < d = 798$ mm:

$V_r = \phi F_v 2/3 A_g [1 - d_n e_c/d(d - d_n)]$
$V_r = (0.90)(2.3 \text{ MPa})2/3(365 \times 798 \text{ mm}^2)[1 - (114 \text{ mm})(200 \text{ mm})/ ((798 \text{ mm})(798 \text{ mm} - 114 \text{ mm}))]$
$\mathbf{V_r = 385\ kN \geq 1.5P_f}$

Now we can compare the various resistances to determine the governing factored load.

Summary:

Moment Resistance:

M_r = 1179 kNm ≥ 3.2P_f
P_f ≤ 368 kN

Longitudinal Shear Resistance:

W_r = 737 kN ≥ 3P_f
P_f ≤ 246 kN (Governs Case b)

Case a) Tension Notch Shear:

F_r = 231 kN ≥ 1.5P_f
P_f ≤ 151 kN (governs Case a)

Case b) Compression Notch Shear:

V_r = 385 kN ≥ 1.5P_f
P_f ≤ 257 kN

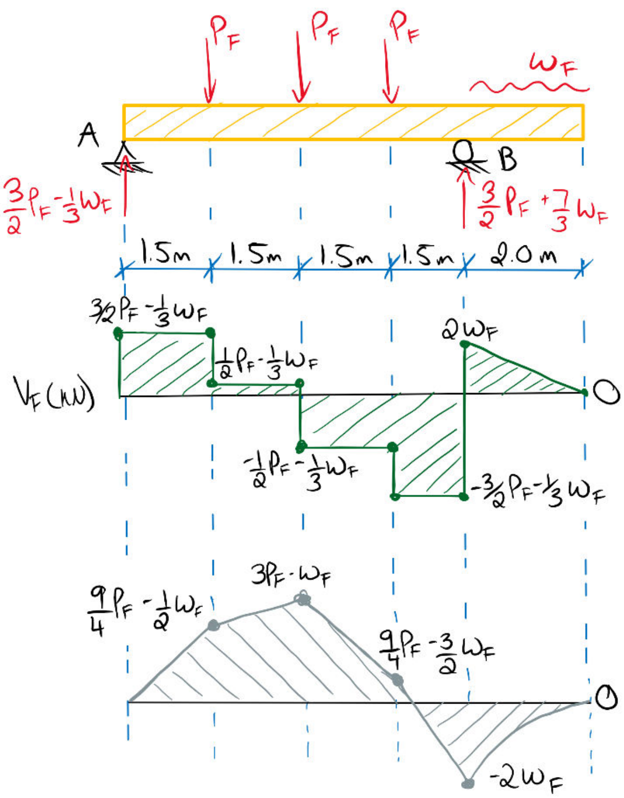
Therefore the governing configuration is Case a) due to tension-side notch shear and the limiting P_f is 151 kN. For Case b), the maximum P_f that can be achieved is 246 kN.

QUESTION 7

The design of this multi-span cantilever beam will first require an analysis of the load combinations to maximize critical effects. Since bearing and deflection can be ignored, the relevant resistances to consider are shear and flexure.

Determining Maximum Factored Loads:

A generalized shear and moment diagram based on variables P_f and w_f can be created to allow easier calculation of factored loads for each combination.



$$\Sigma M_A = 0 = -1.5P_F - 3.0P_F - 4.5P_F + 6P_B - 2w_F \times 7$$

$$P_B = \frac{3}{2}P_F + \frac{7}{3}w_F$$

$$\Sigma M_B = 0 = -2w_F \times 1 + 1.5P_F + 3.0P_F + 4.5P_F - 6P_A$$

$$P_A = \frac{3}{2}P_F - \frac{1}{3}w_F$$

$V_{f,\,max} = -1/3w_f - 1.5P_f$

$M_{f+,\,max} = -w_f + 3P_f$

$M_{f-,\,max} = -2w_f$

Remember that the load duration factor K_D will need to be considered for each load case.

Potential Load Cases for Maximum Factored Positive Moment:

Maximizing P_f and minimizing w_f will produce the maximum positive moment.

1) 1.4D

$P_f = 1.4(20\text{ kN}) = 28\text{ kN}$, $w_f = 1.4(10\text{ kN/m}) = 14\text{ kN/m}$, $K_D = 0.65$

$V_{f,\,max}/K_D = 46.7\text{ kN}/0.65 = 72\text{ kN}$

$M_{f+,\,max}/K_D = 70\text{ kNm}/0.65 = 108\text{ kNm}$

$M_{f-, max}/K_D$ = 28 kNm/0.65 = 28 kNm

2) 1.25D + 1.5L
Note that the snow companion load is omitted on the cantilever to minimize w_f. The snow load will also be omitted from the calculation of K_D since it is not applied.

P_f = 62.5 kN , w_f =12.5 kN/m, K_D = 1.0 – 0.5log(80/75) = 0.99

$V_{f, max}/K_D$ = 97.9 kN/0.99 = 99 kN (governs V_f)
$M_{f+, max}/K_D$ =175 kNm/0.99 = 178 kNm (governs M_{f+})
$M_{f-, max}/K_D$ = 25 kNm/0.99 = 25.2 kNm

Potential Load Cases for Maximum Factored Negative Moment:
3) 0.9D + 1.5S + (1.0L)
Maximizing w_f and minimizing P_f will produce the maximum negative moment. Note the companion live load is omitted on the main span to minimize P_f. The snow load is now considered in K_D but the live load is not.

P_f = 18 kN , w_f =31.5 kN/m, K_D = 1.0 – 0.5log(80/30) = 0.79

$V_{f, max}/K_D$ = -37.5 kN/0.79 = 47.5 kN
$M_{f+, max}/K_D$ = 22.5 kNm/0.79 = 28.5 kNm
$M_{f-, max}/K_D$ = 63 kNm/0.79 = 79.7 kNm (governs M_{f-})

Potential Load Cases for Maximum Factored Shear:
Maximizing all loads normal to the beam will maximize the shear.
2) 1.25D + 1.5L + (1.0S)

Now the companion snow load is considered on the cantilever span to maximize w_f. Note that since the specified dead load is now longer greater than the specific long term loads (since both snow and live load are considered), K_D is taken as the duration factor of the shortest term load per **5.3.2.4**.

P_f = 62.5 kN , w_f = 27.5 kN/m, K_D = 1.15

$V_{f, max}/K_D$ = 103 kN/1.15 = 89.5 kN
$M_{f+, max}/K_D$ = 160 kNm/1.15 = 139 kNm
$M_{f-, max}/K_D$ = 55 kNm/1.15 = 47.8 kNm

Summary of Governing Factored Loads:

V_f = 97.9 kN and K_D = 0.99
M_{f+} = 175 kNm and K_D = 0.99
M_{f-} = 63 kNm and K_D = 0.79

Note that, despite a larger factored shear load in Load Case 2) with the companion snow load, the increased K_D factor (K_D = 1.15) will increase the member's shear resistance such that the same load case without the companion snow load will instead govern the design due to its K_D of 0.99. This dilemma reinforces the importance of the note in 5.3.2.2 which indicates designer judgment should be applied in determination of the duration factor. This example will proceed with the governing case indicated above.

Beam Properties (20f-E SPF glulam):

f_{b+} = 25.6 MPa ; f_{b-} = 19.2 MPa ; f_v = 1.75 MPa ; E = 10300 MPa

K_{sb} = 0.80 ; K_{sf} = 0.85 ; K_{sv} = 0.87 ; K_{SE} = 0.90

To select an approximate section size from the selection tables, we must account for the strength reduction of the K_s and K_D factors. This will be a rough guess since the beam in this configuration does not have standard loading nor a simple span.

$V_{f,\,max}/K_DK_{sv}$ = 97.9 kN/(0.99)(0.87) = 114 kN
$M_{f+,\,max}/K_DK_{sb}$ = 175 kNm/(0.99)(0.80) = 221 kNm
$M_{f-,\,max}/K_DK_{sb}$ = 63 kNm/(0.79)(0.80) = 100 kNm

Assuming the shear resistance may govern the design, try 215x532 20f-E SPF glulam:

$M_{r,\,table}$ = 234 kNm
$V_{r,\,table}$ = 120 kN

<u>Design Check:</u>

<u>Longitudinal Shear:</u>

$Z = 0.215 \times 0.532 \times 8\ m^3 = 0.92\ m^3 < 2.0\ m^3$

Therefore, **7.5.7.2b** can be applied.

$V_r = \phi F_v A_g 2/3 \geq V_f$

$F_v = f_v K_D K_{sv} K_H K_T$ = (1.75 MPa)(0.99)(0.87)(1.0)(1.0) = 1.51 MPa

V_r = (0.90)(1.51 MPa)(215x532 mm^2)2/3 ≥ V_f = 97.9
V_r = 104 kN ≥ 97.9

The longitudinal shear resistance is sufficient for this section.

<u>Moment Resistance:</u>
Per **7.5.6.5.1**, for a beam with multiple points of inflection in the factored moment distribution, the bending size factor must be considered for each segment. In this example, the point of inflection occurs on the roller support. Note that if the load case for the maximum negative moment shifted the point of inflection, the moment resistance calculations would need to be performed separately. However in this case, since the inflections are the same between critical cases, the only accommodation needed to calculate the negative moment resistance is the use of F_{b-} instead of F_{b+}.

$M_{r1} = \phi F_b S K_{zbg} K_x$

$M_{r2} = \phi F_b S K_L K_x$

$F_{b+} = f_{b+} K_D K_H K_{sb} K_T$ = (25.6 MPa)(0.99)(1.0)(0.80)(1.0) = 20.3 MPa
$F_{b-} = f_{b-} K_D K_H K_{sb} K_T$ = (19.2 MPa)(0.79)(1.0)(0.80)(1.0) = 12.1 MPa

$S = bd^2/6 = (215\ mm)(532\ mm)^2/6 = 10.14 \times 10^6\ mm^3$

The lateral stability factor K_L determined in **7.5.6.4** will be the same for all segments. The maximum unsupported length in this example is the 2 m cantilever span. However, since the calculation of L_e varies based on loading condition, let's convince ourselves the cantilever unsupported length will govern over the joist spacing.

Joist Spacing at 1/4 Points: $a = 1.5\ m$, $L_e = 1.54a = 2.31\ m$
Cantilever Length: $l_u = 2\ m$,$L_e = 1.23 l_u = 2.46\ m$ **(governs)**

$C_B = [L_e d/b^2]^{1/2} = [\ (2.46\ m)(0.532\ m)/(0.215\ m)^2\ ]^{1/2} = 5.32 < 10$
$K_L = 1.0$

Beam Segment 1:

$K_{zbg} = [\ 130/b \times 610/d \times 9100/L_1\ ]^{1/10} < 1.3$
$K_{zbg} = [\ 130/(107.5\ mm) \times 610/(532\ mm) \times 9100/(6000\ mm)\ ]^{1/10} < 1.3$
$K_{zbg} = 1.08 < 1.3$

$K_L < K_{zbg}$ therefore M_{r2} will govern this segment.

$M_{r2+} = (0.90)(20.3\ MPa)(10.14 \times 10^6\ mm^3)(1.0)(1.0) \geq M_{f+} = 175\ kNm$
$M_{r2+} = 185\ kNm \geq 175\ kNm$
$M_{r2-} = (0.90)(12.1\ MPa)(10.14 \times 10^6\ mm^3)(1.0)(1.0) \geq M_{f-} = 63\ kNm$
$M_{r2-} = 110\ kNm \geq 63\ kNm$

Beam Segment 2:

$K_{zbg} = [\ 130/(107.5\ mm) \times 610/(532\ mm) \times 9100/(2000\ mm)\ ]^{1/10} < 1.3$
$K_{zbg} = 1.20 < 1.3$

$K_L < K_{zbg}$ therefore M_{r2} will govern this segment. M_{r2} will be the same as the first beam segment.

$M_{r2+} = 185\ kNm \geq 0\ kNm$

$M_{r2-} = 110\ kNm \geq 63\ kNm$

Therefore, the 215x532 section is suitable in flexure and passes all required design checks.