CWC Wood Engineering
Assignment 3 Solutions

QUESTION 1

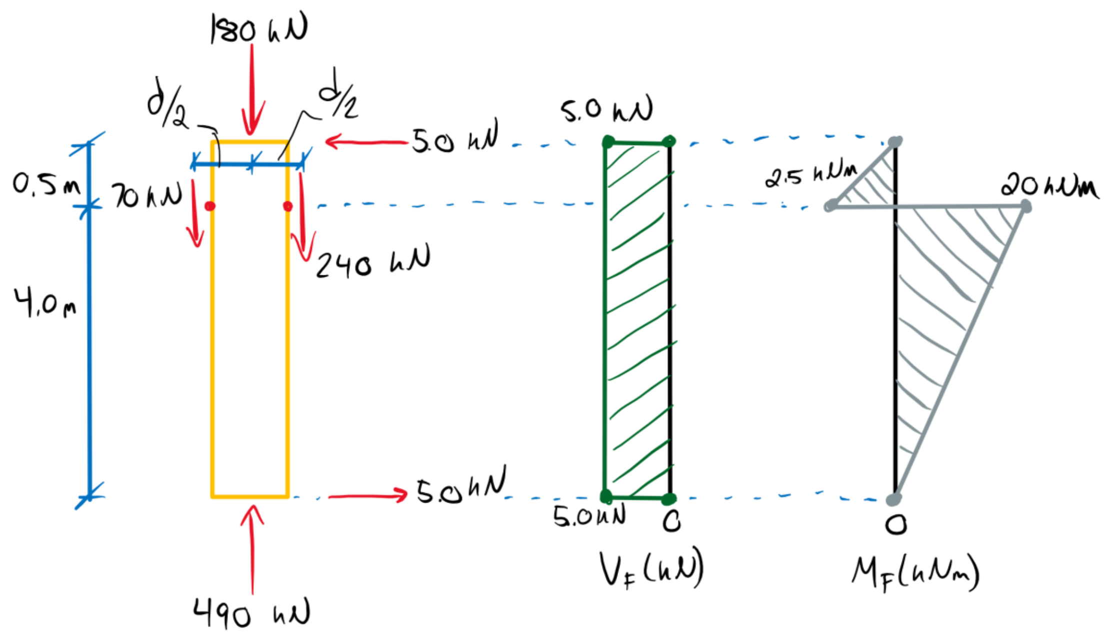
This question requires the analysis of the specified glulam column under the given factored loads using only the tabulated resistance values in O86-14. As there are no compression perpendicular to grain requirements, the values we must seek from the selection tables are the shear resistance, moment resistance, and axial compression resistance. The shear and moment resistances should be taken from the beam selection tables for a 215x266 mm SPF 20f-EX member and the axial compression resistance should be taken from the combined-loading column selection tables for the same member. Note that, while the beam selection tables do not specify the 20f-EX grade, the main difference between 20f-E and 20f-EX is the negative bending stress which is significant in this problem, so the tabulated values apply. Also note that all relevant modification factors are equal to 1.

One parameter that must be checked is the lateral stability factor as the tables assume K_L is taken as 1.0. To confirm this assumption in our problem, we can refer to clause **6.5.4.2.1** which indicates K_L = 1.0 for bending members with depth-to-width ratios of 4:1 or less and with no intermediate support, which our member falls under (1.24:1).

Therefore, from the selection tables:

P_r = 585 kN
M_r = 58.4 kNm (both positive and negative)
V_r = 60.0 kN

The factored loads can be determined considering the d/2 eccentricity from the girder reactions which will induce shear and moment.



P_f = 490 kN < P_r = 585 kN **(Section Passes Axial Comp.)**
M_f = 20 kNm < M_r = 58.4 kNm **(Section Passes Flexure)**
V_f = 5 kN < V_r = 60.0 kN **(Section Passes Shear)**

The section passes the compression, flexure, and shear requirements but it must also meet the requirements of clause **7.5.12** for the interaction axial loads and flexure. The following interaction equation must be satisfied.

$$1.0 \geq (P_f/P_r)^2 + M_f/M_r\,(1 - P_f/P_E)^{-1}$$

where P_E is the Euler buckling load:

$$P_E = \pi^2 E_{05} K_{SE} K_T I / L_e^2$$

E_{05} = 0.87E = 0.87(10300 MPa) = 8961 MPa
$I = bd^3/12 = (215)(266)^3/12\ mm^4 = 337 \times 10^6\ mm^4$

$P_E = \pi^2(8961\ MPa)(1.0)(1.0)(337 \times 10^6\ mm^4)/(1.0 \times 4500\ mm)^2$
P_E = 1472 kN

$1.0 \geq (490/585)^2 + (20/58.4)\,(1 - 490/1472)^{-1}$
$1.0 \geq 0.702 + 0.513$
$1.0 \geq 1.21$ **(Section Fails Interaction)**

Therefore, we must recommend a larger member to meet the interaction requirements. Lets try 215x304 mm SPF 20f-EX. Remember that increasing the member depth will also increase the applied moment due to the eccentricity. Revising M_f gives:

M_f = 25.7 kNm

Shear resistance was already adequate, so we only need to check moment and axial. From the tables:

M_r = 76.3 kNm > M_f = 25.7 kNm
P_r = 662 kN > P_f = 490 kN

$P_E = \pi^2(8961\ MPa)(1.0)(1.0)(503 \times 10^6\ mm^4)/(1.0 \times 4500\ mm)^2$ = 2198 kN

$1.0 \geq (490/662)^2 + (25.7/76.3)\,(1 - 490/2198)^{-1}$
$1.0 \geq 0.98$ **(Section Passes Interaction)**

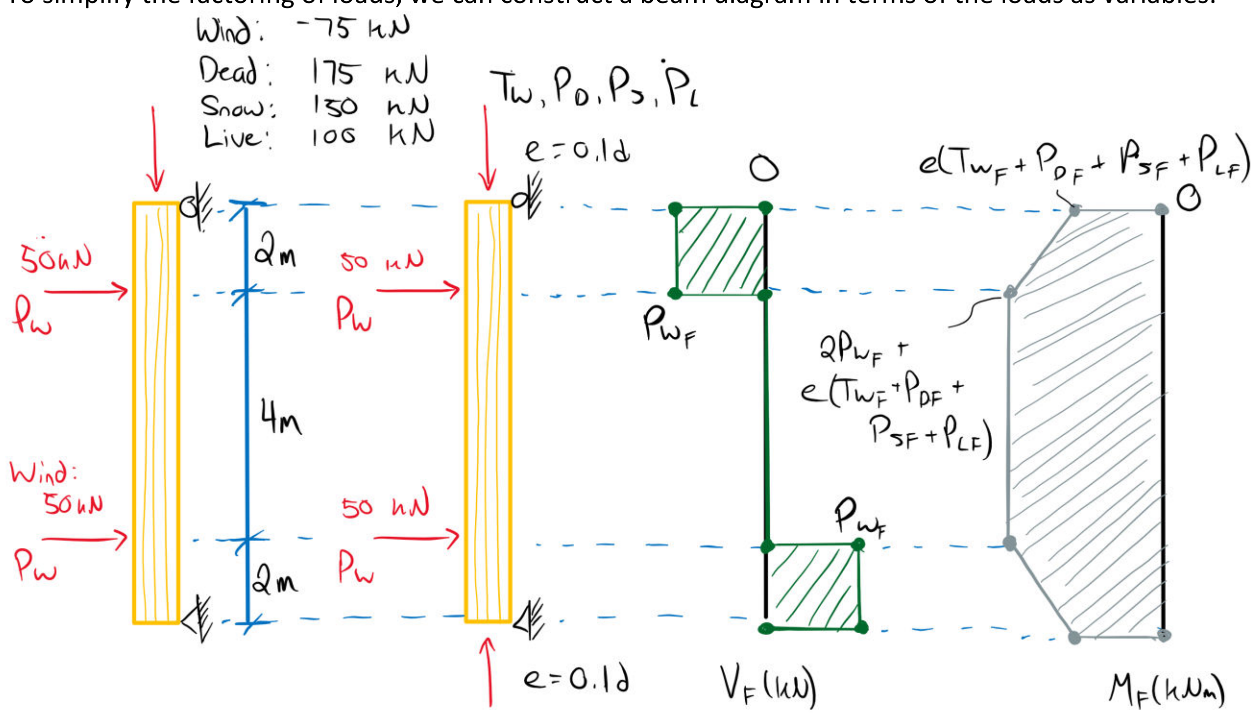
Therefore, the original section was inadequate but 215x304 mm SPF 20f-EX is suitable.

QUESTION 2

Here, we are to conduct a full column design based on multiple specified load types and combined axial and bending loads. The design requires a utilization ratio of 85% or higher, grade 20f-EX, SPF glulam, and a deflection limit of L/180. From Table 7.3:

$f_{b+} = f_{b-} = 25.6$ MPa
$f_v = 1.75$ MPa
$f_c = 25.2$ MPa
$f_{cb} = 25.2$ MPa
$f_{cp} = 5.8$ MPa
$E = 10300$ MPa
$E_{05} = 8961$ MPa
$I = bd^3/12$

To simplify the factoring of loads, we can construct a beam diagram in terms of the loads as variables:



Therefore,

$$M_f = 2P_{w,f} + e(T_{w,f} + P_{D,f} + P_{s,f} + P_{L,f})$$
$$V_f = P_{w,f}$$
(Note the eccentricities are in the same direction at both columns ends so there is no additional shear generated)

We can also estimate the deflection using the superposition of the lateral point load deflection (Δ_1) and the point moment deflections (Δ_2) acting at the eccentric axial loads:

$$\Delta = \Delta_1 + \Delta_2$$

$\Delta = P_w\ a\ (3L^2 - 4a^2)/24EI + ML^2/8EI$

where

$a = 2$ m , $L = 8$ m
$M = e(T_w + P_D + P_s + P_L)$

To approximate a member size, let's assume an eccentricity of 100 mm. This will be a conservative assumption up to a member depth of 1000 mm. Per clause **5.2.4.2** we only need to consider the worst case combination for serviceability requirements. As the lateral wind load contribution is significant, and the wind load uplift is a small portion of the axial load, primarily maximizing P_w should produce the most critical effect, with secondary focus on maximizing the axial force. Therefore, we will proceed with Load Case 4 for the deflection calculation.

Case 4: 1.0D +1.0W + 0.5S
$M = (0.1\ m)(-75 + 175 + 0.5(150)\ )\ kN = 17.5\ kNm$
$P_w = 50\ kN$

Now we can estimate a bending stiffness requirement based on the deflection limit of L/180:

$\Delta = L/180 \geq P_w\ a\ (3L^2 - 4a^2)/24EI + ML^2/8EI$
$EI \geq [P_w\ a\ (3L^2 - 4a^2)/24 + ML^2/8] \times 180/L$
$EI \geq [\ P_w(2\ m)\ (\ 3(8m)^2 - 4(2m)^2\ )/24 + M(8\ m)^2/8\ ] \times 180/(8\ m)$
$EI \geq 330P_w + 180M$ *(Note: here the priority to maximize P_w over M is evident)*
$EI \geq 330(50\ kN) + 180(17.5\ kNm)$
$EI \geq 19650\ kNm^2$ (minimum required stiffness for deflection requirements)

This required stiffness in conjunction with our factored loads can be used to estimate an initial member size.

Factored Loading:

Considering the following load combinations and applying the above equations for M_f , V_f , P_f, we get:

Load Case 1: 1.4 D, $K_D = 0.65$

$P_{D,f} = 1.4D = 1.4(175\ kN) = 245\ kN$

$M_f = (0.1\ m)(245\ kN) = 24.5\ kNm$
$V_f = 0$
$P_f = 245\ kNm$

Load Case 2a: 1.25D + 1.5L + 1.0S, $K_D = 1.0$

$P_{D,f} = 219\ kN$
$P_{L,f} = 150\ kN$
$P_{s,f} = 150\ kN$

$M_f = 51.9\ kNm$

$V_f = 0$
$P_f = 519$ kN

<u>Load Case 2b:</u> 1.25D + 1.5L + 0.4W, $K_D = 1.15$

$P_{D,f} = 219$ kN
$P_{L,f} = 150$ kN
$P_{w,f} = 20$ kN
$T_{w,f} = -30$ kN

$M_f = 73.9$ kNm
$V_f = 20$ kN
$P_f = 339$ kN

<u>Load Case 3a:</u> 1.25D + 1.5S + 1.0L, $K_D = 1.0$

$P_{D,f} = 219$ kN
$P_{L,f} = 100$ kN
$P_{s,f} = 225$ kN

$M_f = 54.4$ kNm
$V_f = 0$ kN
$P_f = 544$ kN

<u>Load Case 3b:</u> 1.25D + 1.5S + 0.4W, $K_D = 1.15$

$P_{D,f} = 219$ kN
$P_{s,f} = 225$ kN
$P_{w,f} = 20$ kN
$T_{w,f} = -30$ kN

$M_f = 81.4$ kNm
$V_f = 20$ kN
$P_f = 414$ kN

<u>Load Case 4:</u> 1.25D + 1.4W + 0.5S, $K_D = 1.15$

$P_{D,f} = 219$ kN
$P_{s,f} = 75$ kN
$P_{w,f} = 70$ kN
$T_{w,f} = -105$ kN

$M_f = 159$ kNm (Governs flexure)
$V_f = 70$ kN (Governs shear)
$P_f = 189$ kN

Recall that, for each different load duration factor, compression resistances must be calculated individually. This will also extend to axial and moment interaction calculations. The governing factored load conditions are summarized below:

Summary:

EI_{min} = 19650 kNm^2
V_f = 70 kN w/ K_D = 1.15
M_f = 159 kNm w/ K_D = 1.15

Interaction Cases:

P_f = 245 kN, M_f = 24.5 kNm w/ K_D = 0.65 (Case 1)
P_f = 519 kN, M_f = 51.9 kNm w/ K_D = 1.0 (Case 2a)
P_f = 339 kN, M_f = 73.9 kNm w/ K_D = 1.15 (Case 2b)
P_f = 544 kN, M_f = 54.4 kNm w/ K_D = 1.0 (Case 3a)
P_f = 414 kN, M_f = 81.4 kNm w/ K_D = 1.15 (Case 3b)
P_f = 189 kN, M_f = 159 kNm w/ K_D = 1.15 (Case 4)

Initial Member Sizing:
Although there is a slew of K_D factors to consider, we can use the member selection tables to get a rough estimate of the required member dimensions. The moment and axial interaction will require some reserve strength in both resistances and should always govern over moment/axial individually. We will not worry about load optimization at this stage.

Try 365x418 mm SPF 20f-EX:

M_r = 245 kNm
V_r = 160 kN
P_r = 1310 kN
EI = 22900 kNm^2

Check axial-moment interaction without considering K_D for rough idea of utilization:

$P_E = \pi^2 E_{05} K_{SE} K_T I / L_e^2$
$P_E = \pi^2(0.87 \times 10300 \text{ MPa})(1.0)(1.0)(365 \times 418^3/12 \text{ mm}^4)/(1.0 \times 8000 \text{ mm})^2$
P_E = 3070 kN

$1.0 \geq (P_f/P_r)^2 + M_f/M_r\,(1 - P_f/P_E)^{-1}$
$1.0 \geq (414/1310)^2 + (81.4/245)\,(1 - (414/3070)\,)^{-1}$
$1.0 \geq 0.10 + 0.384$
$1.0 \geq 0.48$ **(Section Passes but is underutilized)**

Based on our test input, it seems this section may not reach our target utilization of 85% or greater. A 315x418 mm member is very close to the required EI value, let's proceed with a full design calculation for this size.

Full Design Check – 315 x 418 mm SPF 20f-EX:

M_r = 211 kNm

V_r = 138 kN
P_r = 949 kN
EI = 19700 kNm^2

The full design check must consider the respective load duration factors. Keep in mind, compressive resistance parallel to grain will not scale linearly with the load duration factor but the other tabulated values should. Despite this, the question requires full design calculations at this stage but you may use the scaled tabulated values as a sanity check.

Our original assumption assumed an eccentricity of 100 mm. We can update this value to 41.8 mm now that we have a specific member size. This will change the applied factored moment and the deflection requirements.

Deflection:

$\Delta = L/180 \geq P_w\, a\, (3L^2 - 4a^2)/24EI + ML^2/8EI$

Case 4: 1.0D +1.0W + 0.5S (**5.2.4.2**)

M = (0.0418 m)(-75 + 175 + 0.5(150)) kN = 7.32 kNm
P_w = 50 kN

(8 m)/180 ≥ (50 kN)(2m) (3(8 m)^2 – 4 (2 m)^2)/24(19700 kNm^2) + (7.32 kNm)(8 m)^2/8(19700 kNm^2)
0.044 m ≥ (50 kN)(2m) (3(8 m)^2 – 4 (2 m)^2)/24(19700 kNm^2) + (7.32 kNm)(8 m)^2/8(19700 kNm^2)
0.044 m ≥ 0.0372 m + 0.0029 m
0.044 m ≥ 0.040 m **(Section Passes Deflection Requirements)**

Shear (**7.5.7**):

Z = 8 m x 0.315 m x 0.418 m = 1.05 m^3 < 2.0 m^3

We can proceed with the simplified shear calculation for beam volumes less than 2.0 m^3 (**7.5.7.2**):

$V_r = \phi F_v A_g\, 2/3$

$F_v = f_v K_D K_H K_{sv} K_T$ = (1.75 MPa)(1.15)(1)(1)(1) = 2.01 MPa

V_r = (0.90)(2.01 MPa)(315x418 mm^2)2/3
V_r = 159 kN ≥ V_f = 70 kN **(Section Passes Shear Requirements)**

Flexure (**7.5.6.5**):
We require a moment resistance value for each of the K_D values we must check in combined axial and bending. The factors K_L and K_{zbg} will determine the governing moment resistance per **7.5.6.5.1**:

$K_{zbg} = (130/157.5 \times 610/418 \times 9100/8000)^{0.1} \leq 1.3$
K_{zbg} = 1.03

K_L = 1.0 (for members width depth : width ratios less than 4:1 per **6.5.4.2.1**)

Therefore, M_{r2} will govern the moment resistance as $K_L < K_{zbg}$.

$M_{r2} = \phi F_b S K_x K_L$

$F_b = f_b K_D K_H K_{sb} K_T$ = (25.6 MPa)K_D(1)(1)(1) = 25.6K_D MPa

$S = bd^2/6 = 9.17\text{x}10^6\ mm^3$

M_{r2} = (0.9)(25.6K_D MPa)(9.17x10^6 mm^3)(1)(1)
M_{r2} = 211.3K_D kNm

Comparing to the updated factored moments based on the new eccentricity of 0.1d = 41.8 mm:

M_r = 137 kNm > M_f = 10.2 kNm w/ K_D = 0.65
M_r = 243 kNm > M_f = 148 kNm w/ K_D = 1.15 **(Governs)**
M_r = 211 kNm > M_f = 22.7 kNm w/ K_D = 1.0

(Section Passes Flexural Requirements)

Axial Compression (**7.5.8**):
Similarly, we must calculate an axial compression resistance for each relevant K_D.

$C_C = 8\ m/0.315\ m = 25.4 \le 50$
$P_r = \phi F_c A K_{zcg} K_C$
$F_c = f_c K_D K_H K_{sc} K_T$ = (25.2 MPa)K_D(1)(1)(1) = 25.2K_D MPa
$K_{zcg} = 0.68(Z)^{-0.13} = 0.68(1.05\ m^3)^{-0.13} = 0.68 \le 1.0$

K_D = 0.65:
F_c = 16.4 MPa
$K_C = [1.0 + F_C K_{zcg} C_C^3 / 35 E_{05} K_{se} K_T]^{-1} = [1.0 + (16.4\ MPa)(0.68)(25.4)^3/35(8961\ MPa)(1)(1)]^{-1}$
K_C =0.63
P_r = (0.8)(16.4 MPa)(315x418 mm^2)(0.68)(0.63)
P_r = 740 > P_f = 245 kN

K_D = 1.15:
F_c =29.0 MPa
K_C = 0.49
P_r = 1018 kN > P_f = 544 kN

K_D = 1.0:
F_c =25.2 MPa
K_C = 0.53
P_r = 957 kN > P_f = 414 kN
(Section Passes Axial Compression Requirements)

Combined Axial and Bending (**7.5.12**):

The interaction equation must be checked for each load case, not just per value of K_D. This is because it may not be obvious which combination of axial and bending loads is most critical.

$P_E = \pi^2 E_{05} K_{SE} K_T I / L_e^2$
$P_E = \pi^2 (8961\ MPa)(1.0)(1.0)(315x418^3/12\ mm^4)/(1.0\ x\ 8000\ mm)^2$
$P_E = 2649\ kN$

Case 1: $P_r = 740\ kN$, $M_r = 137\ kNm$, $P_f = 245\ kN$, $M_f = 10.2\ kNm$ w/ $K_D = 0.65$

$1.0 \geq (P_f/P_r)^2 + M_f/M_r\ (1 - P_f/P_E)^{-1}$
$1.0 \geq (245/740)^2 + (10.2/137)\ (1 - (245/2649)\ )^{-1}$
$1.0 \geq 0.110 + 0.082$
$1.0 \geq 0.19$ **(Section Passes)**

Case 2a: $P_r = 957\ kN$, $M_r = 211\ kNm$, $P_f = 519\ kN$, $M_f = 21.7\ kNm$ w/ $K_D = 1.0$

$1.0 \geq 0.42$ **(Section Passes)**

Case 2b: $P_r = 1018\ kN$, $M_r = 243\ kNm$, $P_f = 339\ kN$, $M_f = 54.2\ kNm$ w/ $K_D = 1.15$

$1.0 \geq 0.37$ **(Section Passes)**

Case 3a: $P_r = 957\ kN$, $M_r = 211\ kNm$, $P_f = 544\ kN$, $M_f = 22.7\ kNm$ w/ $K_D = 1.0$

$1.0 \geq 0.46$ **(Section Passes)**

Case 3b: $P_r = 1018\ kN$, $M_r = 243\ kNm$, $P_f = 414\ kN$, $M_f = 57.3\ kNm$ w/ $K_D = 1.15$

$1.0 \geq 0.44$ **(Section Passes)**

Case 4: $P_r = 1018\ kN$, $M_r = 243\ kNm$, $P_f = 189\ kN$, $M_f = 148\ kNm$ w/ $K_D = 1.50$

$1.0 \geq 0.84$ **(Section Passes - GOVERNS)**

Summary/Utilization:

Deflection:

$\Delta = 0.040\ m < 0.044\ m$ 0.040/0.044 x 100 = 91% **(Governs Utilization)**

Shear:

$V_r = 159\ kN \geq V_f = 70\ kN$ 70/159 x 100 = 44%

Moment:

$M_r = 243\ kNm > M_f = 148\ kNm$ 148/243 x 100 = 61%

Axial Comp.:

P_r = 1018 kN > P_f = 544 kN 544/1018 x 100 = 53%

Axial and Moment Interaction:

1.0 ≥ 0.82 84%

Therefore, the utilization of the 315x418 mm SPF 20f-EX glulam member is 91%, governed by the deflection requirements. This member is suitable for the given specified loads.

QUESTION 3

This question requires the determination of the maximum lateral wind load based on the specified axial loads. The stud wall consists of 38x140 mm SPF No. 2 members at 500 mm spacing with 10 mm plywood sheathing and 2 inch common nails spaced at 150 mm.

Member Properties (38x140 mm SPF No. 2)
Structural Joist and Plank (**Table 6.2.2.1, Table 6.3.1A**):

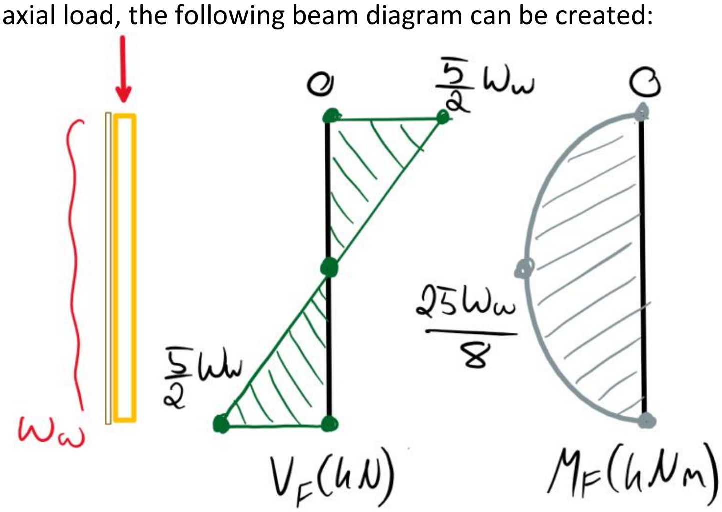
f_b = 11.8 MPa
f_v = 1.5 MPa
f_c = 11.5 MPa
E = 9500 MPa
E_{05} = 6500 MPa

Based on the requirements of clause **6.4.4.2** and **Table 6.4.5** the following modification factors are relevant:

K_{zb} = 1.4
K_{zv} = 1.4
K_{Hb} = 1.4
K_{Hc} = 1.10
K_{Hv} = 1.10

The question also indicates to only consider load cases duration factors equal to 1.15. There are no treatment or service conditions to consider.

The maximum factored loads can be determined in terms of the unknown factored wind load $w_{w,f}$. With no eccentricity in the axial load, the following beam diagram can be created:



$M_f = 25\ w_{w,f}/8$
$V_f = 5\ w_{w,f}/2$

The factored axial load is dependent on the load case. The following load cases that yield $K_D = 1.15$ must be considered. This implies load cases including wind as a primary or companion load need consideration.

Load Case 2: 1.25D + 1.5L + 0.4W $\quad$ $K_D = 1.15$

P_f = 1.25(6 kN/m) + 1.5(10 kN/m) = 22.5 kN/m **(Governs Critical $w_{w,f}$)**

Load Case 4a: 1.25D + 1.4W + 0.5L $\quad$ $K_D = 1.15$

P_f = 1.25(6 kN/m) + 0.5(10 kN/m) = 6.25 kN/m

We are interested in the most critical value of $w_{w,f}$ which will occur for the largest axial (thus requiring a smaller lateral load to reach the critical moment-axial interaction). It is for this reason that Case 4 with no companion load is not considered.

Shear Resistance (**6.5.5.2**)

$V_r = \phi F_v A_n K_{zv} 2/3$

$F_v = f_v K_D K_{Hv} K_{sv} K_T$ = (1.5 MPa)(1.15)(1.4)(1)(1) = 2.4 MPa

V_r = (0.9)(2.4 MPa)(38x140 mm^2)(1.4) x 2/3

$\mathbf{V_r = 10.7\ kN \geq V_f = 5\ w_{w,f}/2}$

Moment Resistance (**6.5.4**)

$M_r = \phi F_b S K_{zb} K_L$

$F_b = f_b K_D K_{Hb} K_{sb} K_T$ = (11.8 MPa)(1.15)(1.40)(1) = 19.0 MPa

$S = bd^2/6 = (38)(140)^2/6\ mm^3 = 0.124 \times 10^6\ mm^3$

For b/d = 3.7 (**6.5.4.2.1**),

$K_L = 1.0$

M_r = (0.9)(19.0 MPa)(0.124 x 10^6 mm^3)(1.4)(1.0)

$\mathbf{M_r = 3.0\ kNm \geq M_f = 25\ w_{w,f}/8}$

Axial Compression (**6.5.6.2.3)**

Based on clause **6.5.6.5**, the depth of the stud members can be used to calculate the slenderness ratio since it is sheathed. Pin-pin support is assumed.

$C_{Cx} = C_{Cy} = L_e/d$ = (5.0 m)/(0.14 m) = 35.7 < 50

$P_r = \phi F_c A K_{zc} K_c$

$F_c = f_c K_D K_{Hc} K_{sc} K_T$ = (11.5 MPa)(1.15)(1.10)(1)(1) = 14.5 MPa

$K_{zc} = 6.3(dL)^{-0.13} = 6.3[\ (140)(5000)\ ]^{-0.13} = 1.10 < 1.3$

$K_c = [1.0 + F_C K_{zc} C_C^3 / 35 E_{05} K_{se} K_T\ ]^{-1} = [1.0 + (14.5\ MPa)(1.10)(35.7)^3/35(6500\ MPa)(1)(1)]^{-1}$

$K_c = 0.24$

$P_r = (0.80)(14.5\ \text{MPa})(38\text{x}140\ \text{mm})(1.10)(0.24)$
$\mathbf{P_r = 16.3\ kN \geq P_f = 11.25\ kN}$

Combined Axial and Moment (**6.5.10**)

$P_E = \pi^2 E_{05} K_{SE} K_T I / L_e^2$
$P_E = \pi^2 (6500\ \text{MPa})(1.0)(1.0)(38\text{x}140^3/12\ \text{mm}^4)/(5000\ \text{mm})^2$
$P_E = 22.3\ \text{kN}$

The interaction equation must be solved for M_f to determine the limit on the lateral load.

$1.0 \geq (P_f/P_r)^2 + M_f/M_r\ (1 - P_f/P_E)^{-1}$
$M_f \leq [1 - (P_f/P_r)^2]\ \text{x}\ [1 - P_f/P_E]\ \text{x}\ M_r$
$M_f \leq [1 - (11.25/16.3)^2]\ \text{x}\ [1 - (11.25)/(22.3)]\ \text{x}\ (3.0\ \text{kNm})$
$\mathbf{M_f \leq 0.78\ kNm}$

Summary
Shear:

$V_r = 10.7\ \text{kN} \geq V_f = 5\ w_{w,f}/2$
$\mathbf{w_{w,f} \leq 4.3\ kN/m}$

Moment:

$M_r = 3.0\ \text{kNm} \geq M_f = 25\ w_{w,f}/8$
$\mathbf{w_{w,f} \leq 0.96\ kN/m}$

Axial + Moment:

$M_f \leq 0.78\ \text{kNm} = 25\ w_{w,f}/8$
$\mathbf{w_{w,f} \leq 0.25\ kN/m}$ (**Governs**)

This governing value of $w_{w,f}$ should be compared to the value in the stud wall selection tables (SPF No. 2 38x140 mm studs with a length of 5 m and no eccentricity). We can interpolate based on the factored axial load to determine the maximum lateral wind load.

Interpolating with $P_f = 11.25$ kN/m
$(w_{w,f} - 0.239) / (0.344 - 0.239) = (11.25 - 11.3) / (9.71 - 11.3)$
$\mathbf{w_{w,f} = 0.24\ kN/m}$

This tabulated value is very close to the calculated limit on $w_{w,f}$. For this problem, as soon as P_f was known $w_{w,f}$ could have been determined.

QUESTION 4

Part A:

This first part of this question requires the manual calculation of the effective bending stiffnesses and shear rigidities for the CLT panels in question. This is done through clause **8.4.3.2**. Although the calculations should be done by hand, the final answers can be compared to **Table 2.12** in Volume 1 of the CWC Wood Design Manual.

CLT Properties (5-ply Grade E2 – **Table 8.2.4** in **O86-14**)

$t = 35$ mm
$b_x = 2.4$ m
$b_y = 10$ m

Longitudinal Layers (layers 1, 3, 5):

$f_b = 23.9$ MPa
$f_s = 0.63$ MPa
$E_x = 10300$ MPa
$E_{x,\perp} = E_x/30 = 343$ MPa
$G_x = E_x/16 = 644$ MPa
$G_{x,\perp} = G_x/10 = 64.4$ MPa

Transverse Layers (layers 2, 4):

$f_b = 4.6$ MPa
$f_s = 0.63$ MPa
$E_y = 10000$ MPa
$E_{y,\perp} = E_y/30 = 333$ MPa
$G_y = E_y/16 = 625$ MPa
$G_{y,\perp} = G_y/10 = 62.5$ MPa

For the effective bending stiffness in the longitudinal strength direction:

$EI_{eff,x} = \Sigma Ebt^3/12 + \Sigma Ebtz^2$

Note that this summation will include terms two terms for layers 1 and 5, one term for layer 3, and two terms for layers 2 and 4. The transverse layers contribute to the bending stiffness. The z variable refers to the distance to the centroid of each layer from the centroid of the full panel cross section. Therefore layers 2 and 4, as well as 1 and 5, will have the same value for z. Layer 3 has z =0.

$EI_{eff,x} = [E_x b_x t^3/12]_{Layer\ 3} + 2\,[E_{y,\perp} b_x t^3/12 + E_{y,\perp} b_x t\, z_2^2]_{Layers\ 2,\ 4} + 2[E_x b_x t^3/12 + E_x b_x t\, z_5^2]_{Layers\ 1,\ 5}$
$EI_{eff,x} = [88.32 \text{ kNm}^2] + [74.24 \text{ kNm}^2] + [8655.61 \text{ kNm}^2]$
$EI_{eff,x} = 8818 \text{ kNm}^2$

From **Table 2.12 in Vol. 1**:

$EI_{eff,x} = 3670 \times 10^9 \text{ Nmm}^2/\text{m} \times 2.4 \text{ m}$
$EI_{eff,x} = 8808 \text{ kNm}^2$

For the effective bending stiffness in the transverse direction:

$EI_{eff,\,y} = \Sigma Ebt^3/12 + \Sigma Ebtz^2$

The transverse bending stiffness does not utilize the extreme layers (Layers 1 and 5) and therefore will only have two terms for layers 2 and 4, and one term for layer 3.

$EI_{eff,\,y} = [E_{x,\perp}\ b_y t^3/12]_{Layer\ 3} + 2\ [E_y\ b_y t^3/12 + E_y\ b_y t\ z_2^2]_{Layers\ 2,\ 4}$
$EI_{eff,\,y} = 12.26\ kNm^2 + 9289.58\ kNm^2$
$\mathbf{EI_{eff,\,y} = 9302\ kNm^2}$

From **Table 2.12 in Vol. 1**:

$EI_{eff,\,y} = 930\ x10^9\ Nmm^2\ /m\ x\ 10.0\ m$
$\mathbf{EI_{eff,\,y} = 9300\ kNm^2}$

For the longitudinal in-plane shear rigidity:

$GA_{eff,\,x} = (h - t_1/2 - t_5/2)^2\ /\ [\ t_1/2G_1b_x + \Sigma\ t_i/G_ib_x + t_5/2G_5b_x\ ]$
$GA_{eff,\,x} = (h - t_1/2 - t_5/2)^2\ /\ [\ t_1/2G_xb_{x\ (Layer\ 1)} + 2(t_2/G_{y,\perp}\ b_x)_{\ (Layers\ 2,\ 4)} + t_3/G_xb_{x\ (Layer\ 3)} + t_5/2G_xb_{x\ (Layer\ 5)}\ ]$
$GA_{eff,\,x} = (140\ mm)^2\ /\ [5.120\ x\ 10^{-4}\ MPa^{-1}]$
$\mathbf{GA_{eff,\,x} = 38281\ kN}$

From **Table 2.12 in Vol. 1**:

$GA_{eff,\,x} = 16.0\ x10^6\ N/m\ x\ 2.4\ m$
$\mathbf{GA_{eff,\,x} = 38400\ kN}$

For the tranvserse in-plane shear rigidity all layers are considered:

$GA_{eff,\,y} = (h - t_1/2 - t_5/2)^2\ /\ [\ t_1/2G_1b_y + \Sigma\ t_i/G_ib_y + t_5/2G_5b_y\ ]$
$GA_{eff,\,y} = (h - t_1/2 - t_5/2)^2\ /\ [\ t_1/2G_{x,\perp}b_{y\ (Layer\ 1)} + 2(t_2/G_y b_y)_{\ (Layers\ 2,\ 4)} + t_3/G_{x,\perp}b_{y\ (Layer\ 3)} + t_5/2G_{x,\perp}b_{y\ (Layer\ 5)}$
$GA_{eff,\,y} = (140\ mm)^2\ /\ [1.199\ x\ 10^{-4}\ MPa^{-1}]$
$\mathbf{GA_{eff,\,y} = 163470\ kN}$

From **Table 2.12 in Vol. 1**:

$GA_{eff,\,x} = 16.3\ x10^6\ N/m\ x\ 10.0\ m$
$\mathbf{GA_{eff,\,x} = 163000\ kN}$

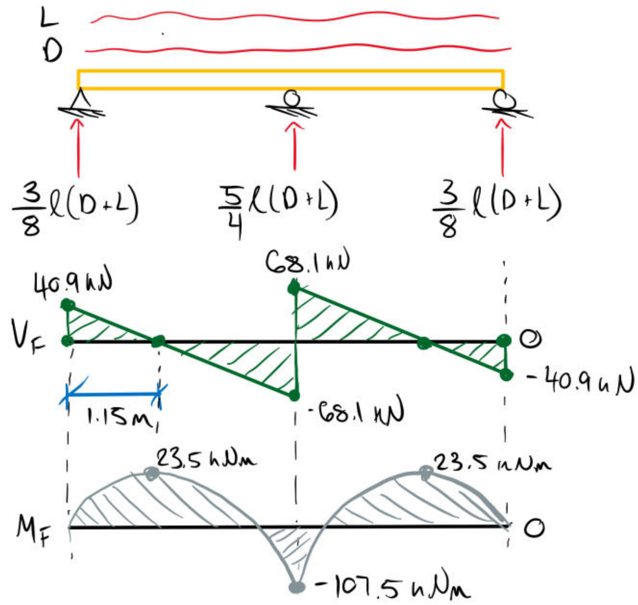
All calculated stiffness and rigidity values match well with the tabulated values.

Part B:
Now we must perform ULS design checks for the CLT panel based on the specified loading. We must consider the following load cases for dead and live loads:

Load Case 2a: 1.25D + 1.5L $K_D = 1.0$

$w_f = 1.25(1.5\ kPa\ x\ 2.4\ m) + 1.5(4.8\ kPa\ x\ 2.4\ m) = 21.8\ kN/m$

We will consider this loading case over the full span of the panels.



M_{f+} = 23.5 kNm

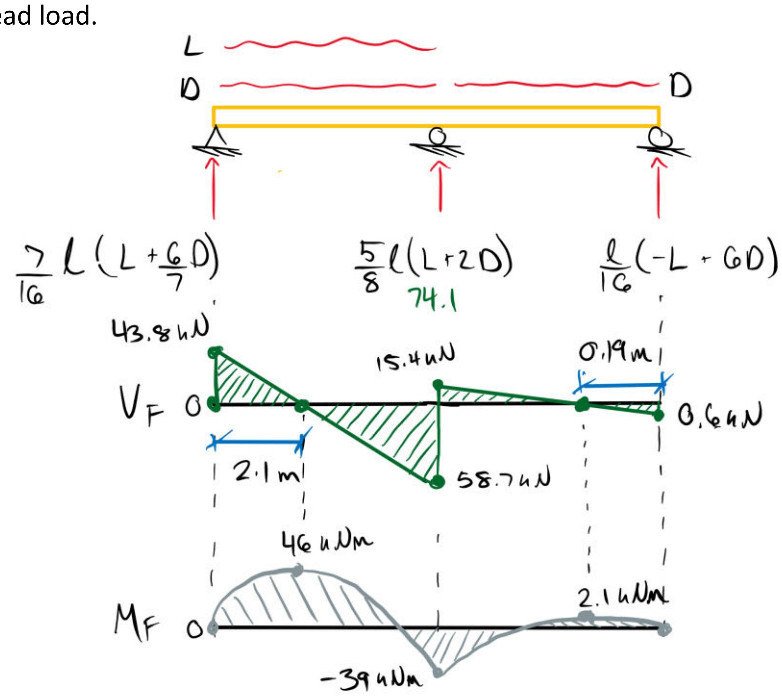
M_{f-} = 107.5 kNm (Governs)

V_f = 68.1 kN (Governs)

<u>Load Case 2b:</u> 0.9D + 1.5L K_D = 1.0

w_f = 0.9(1.5 kPa x 2.4 m) + 1.5(4.8 kPa x 2.4 m) = 3.2 kN/m + 17.3 kN/m

To maximize the factored positive moment, we will omit live load from half of the CLT span and apply the 0.9 factor to the dead load.



M_{f+} = 46 kNm (Governs)
M_{f-} = 39 kNm
V_f = 58.7 kN

Factored Loads:

M_{f+} = 46 kNm
M_{f-} = 107.5 kNm
V_f = 68.1 kN
$V_f(d)$ = 65.7 kN (at a distance d from support)

Moment Resistance (**8.4.3**)
For the longitudinal major strength direction:

$M_r = \phi F_b S_{eff,\,x} K_{rb,\,x}$

$K_{rb,\,x}$ = 0.85 Calibration factor for the major strength direction

$F_b = f_{b,x} K_D K_H K_{sb} K_T$
F_b = (23.9 MPa)(1)(1)(1)(1) = 23.9 MPa

$S_{eff,x} = EI_{eff,\,x}/E_x \times 2/h$
$S_{eff,x}$ = (8808 x 10^9 Nmm^2)/(10300 MPa) x 2/(175 mm)
$S_{eff,x}$ = 9.085 x 10^6 mm^3

M_r = (0.9)(23.9 MPa)(9.085 x 10^6 mm^3)(0.85)
M_r =166 kNm > M_f = 107.5 kNm (**Section Passes Flexural Check**).

Shear Resistance (**8.4.4**)
For the major strength direction:

$V_r = \phi F_s A_{g,\,zx}\ 2/3$

$F_s = f_s K_D K_H K_{sv} K_T$ = (0.63 MPa)(1)(1)(1)(1) = 0.63 MPa

V_r = (0.90)(0.63 MPa)(2400x175 mm^2)2/3
V_r = 159 kN > V_f = 65.7 kN (Section Passes Shear Check)

Therefore, the CLT panel passes all required ULS checks.

Part C:
Annex **A.8.5.2** and **A.8.5.3** provide calculations for the SLS (deflection and vibration) assessment of CLT members.

For a deflection limit of L/180 and a single span CLT panel exposed to a distributed load:

$\Delta = \Delta_{st} + \Delta_{LT} K_{creep}$

where the long term load effects of dead load are multiplied by a creep factor.

$\Delta = [5/384\ w_L L4/EI_{eff,x} + 1/8\ w_L L^2 \kappa/GA_{eff,x}]_{Live\ Load} + K_{creep}[5/384\ w_D L^4/EI_{eff,x} + 1/8\ w_D L^2 \kappa/GA_{eff,x}]_{Dead\ Load}$

$\kappa = 1.2$ Shear deformation form factor for rectangular cross-sections.

$\Delta = [\ 5/384\ (11.52)(5000)^4/(8188 \times 10^9) + 1/8\ (11.52)(5000)^2(1.2)/(0.16 \times 10^9)\ ]_{Live\ Load} +$
$(2.0)\ [\ 5/384\ (3.6)(5000)^4/(8188 \times 10^9) + 1/8\ (3.6)(5000)^2(1.2)/(0.16 \times 10^9)\ ]_{Dead\ Load}$

$\Delta = 14.15\ mm + 3.64\ mm$

$\Delta = 17.8\ mm < L/180 = 5000/180\ mm = 27.8\ mm$ (**Section Passes Deflection Check**)

For the limiting span based on vibration requirements

$L_{max} = 0.11\ (EI_{eff,x}/10^6)^{0.29} / m^{0.12}$

$m = 420\ kg/m^3 \times 1.0\ m \times 0.175\ m = 73.5\ kg/m$ Linear mass of CLT for a 1 m wide panel

$L_{max} = 0.11\ (8188 \times 10^9 / 10^6)^{0.29} / (73.5)^{0.12}$

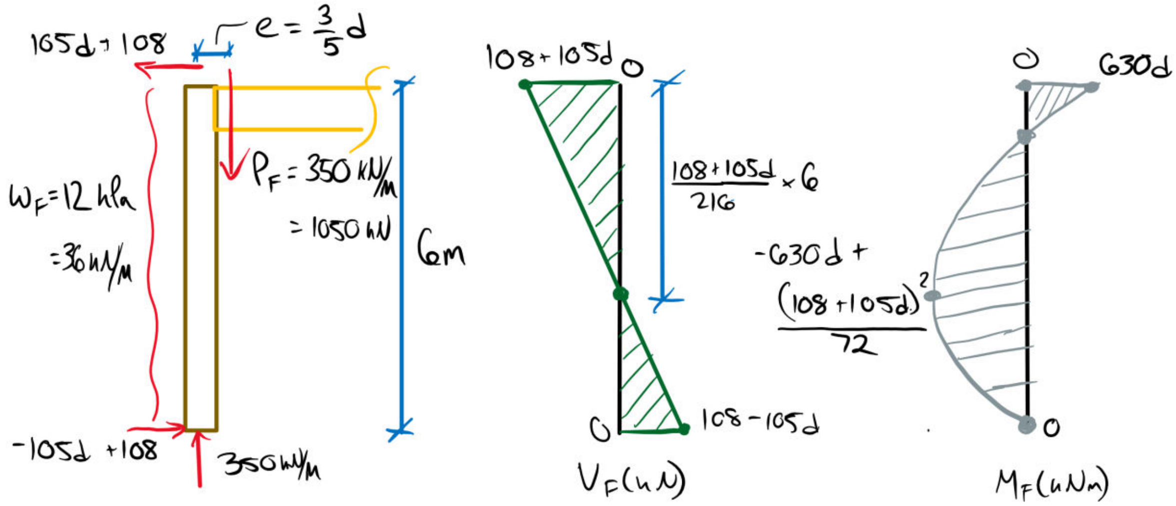
$L_{max} = 6.6\ m < L = 5\ m$ **(Section O.K for Vibration)**

Therefore based on Annex **A.8**, this section meets the SLS deflection and vibration requirements.

QUESTION 5

This question requires the design a CLT bearing wall subjected to axial and lateral loads. The panel width is 3 m and a CLT grade of E1 is required. The applied loads are factored so load cases need not be considered. The load duration factor is assumed to be 1.0.

We can use loading diagrams to determine the maximum factored loads. The axial load eccentricity will vary depending on the final design which will affect the maximum moment and shear values. The eccentricity is only applied to the top axial loads.



M_{f+} = -630d + (108 + 105d)^2/72
M_{f-} = -630d
V_f =108 + 105d
P_f = 1050 kN

Since K_D = 1.0 and e > d/2, the tabulated CLT resistances will not be conservative. With this knowledge, we can see that likely a 7-ply or 9-ply CLT panel will be required and a 5-ply will be insufficient. Let's first try a 7-ply since we are after the smallest number of plies.

<u>CLT Grade E1 – 7 ply:</u>

f_c = 19.3 MPa
f_s = 0.50 MPa
f_b = 28.2 MPa
d = 245 mm
E = 11700 MPa
E_{05} =0.87E = 9594 MPa
$EI_{eff,\,x}$ = 30900 kNm^2
$EI_{eff,\,y}$ = 9660 kNm^2
$GA_{eff,\,x}$ = 65700 kN
$GA_{eff,\,y}$ = 81600 kN

The factored loads will be:

M_{f+} = 94 kNm
M_{f-} = 154 kNm
V_f =134 kN
P_f = 1050 kN

Shear Resistance (**8.4.4**)
For the major strength direction:

$V_r = \phi F_s A_{g,\, zx}\, 2/3$

$F_s = f_s K_D K_H K_{sv} K_T$ = (0.50 MPa)(1)(1)(1)(1) = 0.50 MPa

V_r = (0.90)(0.50 MPa)(3000 x 245 mm^2)2/3
V_r = 220 kN > V_f = 134 kN (Section Passes Shear Check)

Moment Resistance (**8.4.3**)
For the longitudinal major strength direction:

$M_r = \phi F_b S_{eff,\, x} K_{rb,\, x}$

$K_{rb,\, x}$ = 0.85 Calibration factor for the major strength direction

$F_b = f_{b,x}\, K_D K_H K_{sb} K_T$
F_b = (28.2 MPa)(1)(1)(1)(1) = 28.2 MPa

$S_{eff,x} = EI_{eff,\, x}/E_x \times 2/h$
$S_{eff,x}$ = (30900 x 10^9 Nmm^2)/(11700 MPa) x 2/(245 mm)
$S_{eff,x}$ = 21.56 x 10^6 mm^3

M_r = (0.9)(28.2 MPa)(21.56x 10^6 mm^3)(0.85)
M_r =465 kNm > M_f = 154 kNm (Section Passes Flexural Check).

Axial Compression Resistance (**8.4.5.4.2**)
In axial compression, the transverse CLT layers are not considered and the effective moment of inertia must be calculated independently from the tabulated values. For the longitudinal major strength direction

$P_r = \phi F_C A_{eff} K_{zc} K_C$

$F_c = f_c K_D K_H K_{sc} K_T$ = (19.3 MPa)(1)(1)(1)(1) = 19.3 MPa
$K_{zc} = 6.3(12^{0.5}\, r_{eff} L)^{-0.13} \leq 1.3$
$C_C = L_e/12^{0.5}\, r_{eff}$

$r_{eff} = (I_{eff}/A_{eff})^{0.5}$

A_{eff} = 4 x (35 mm x 3000 mm) = 420000 mm^2

$I_{eff} = \Sigma (bt^3/12 + btz^2)$ This is the parallel axis theorem from Mechanics class!

$I_{eff} = 4 \times (bt^3/12) + bt(z_1^2 + z_3^2 + z_5^2 + z_7^2)$

$I_{eff} = 4 \times (3000)(35)^3/12\ mm^4 + (3000)(35)[\ (3 \times 35)^2 + (35)^2 + (35)^2 + (3 \times 35)^2\ ]\ mm^4$

$I_{eff} = 42.875 \times 10^6\ mm^4 + 2572.5 \times 10^6\ mm^4$

$I_{eff} = 2615 \times 10^6\ mm^4$

$r_{eff} = [\ (2615 \times 10^6\ mm^4/(420000\ mm^2)\ ]^{0.5} = 78.8\ mm$

$C_C = (6000\ mm)/12^{0.5}(78.8\ mm) = 22.0 < 43.0$ **(slenderness ratio O.K)**

$K_{zc} = 6.3(12^{0.5}\ (78.8)(6000)\ )^{-0.13} = 0.98 \leq 1.3$

$K_C = [1.0 + F_C K_{zc} C_C^3\ /\ 35 E_{05} K_{se} K_T\ ]^{-1} = [1.0 + (19.3\ MPa)(0.98)(22.0)^3/35(9594\ MPa)(1)(1)]^{-1}$

$K_C = 0.625$

$P_r = (0.80)(19.3\ MPa)(420000\ mm^2)(0.98)(0.625)$

Pr =3972 > Pf = 1050 kN (Section Passes Axial Comp. Check)

Combined Axial and Moment (**8.4.6**)

$P_f/P_r + M_f/M_r\ (1 - P_f/P_{E,v})^{-1} \leq 1.0$

$P_{E,v} = P_E/(1 + \kappa P_E/GA_{eff,x})$ (Euler buckling considering shear deformations)

$P_E = \pi^2 E_{05} K_{SE} K_T I_{eff}/L_e^2 = \pi^2(9594\ MPa)(1)(1)(2605 \times 10^6\ mm^4)/(6000\ mm)^2 = 6852\ kN$

$P_{E,v} = (6852\ kN)/(1 + (1.2)(6852\ kN)/(65700\ kN) = 6090\ kN$

$1.0 \geq (1050)/(3972) + (154)/(465)(1 - 1050/6090)^{-1}$

$1.0 \geq 0.264 + 0.400$

1.0 ≥ 0.66 (Section Passes Moment + Axial Interaction)

Therefore, 7-ply grade E1 CLY is the smallest number of plies for this application. Need not check 5 plies since the tabulated values are insufficient and unconservative for e > d/2 and $K_D = 1.0$.