Lecture #3 – Design of Axially Loaded Members

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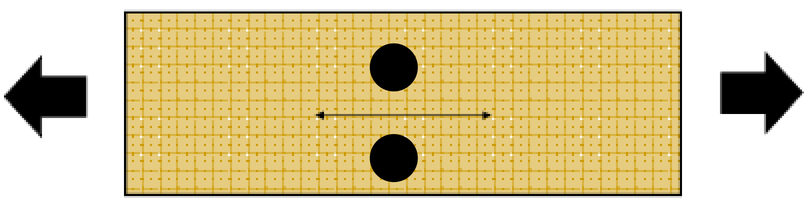
Axially Loaded Members

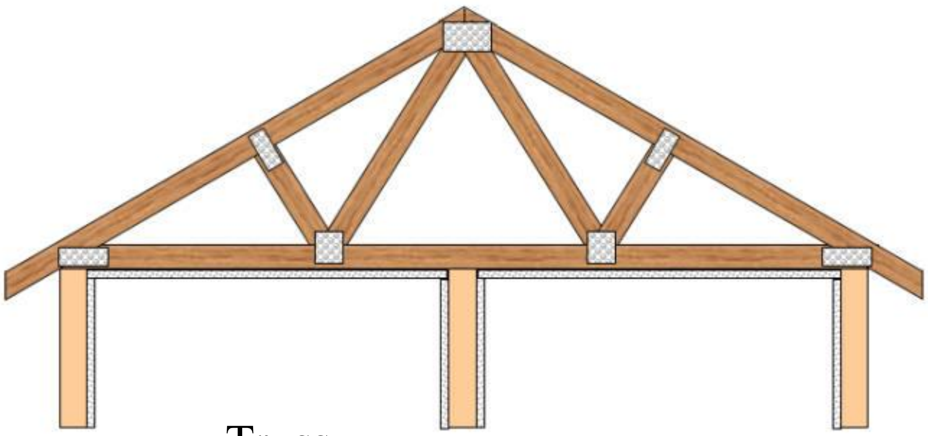
- Tension members
- Compression members
- Members subjected to combined bending and axial force

DESIGN OF TENSION MEMBERS

Loaded parallel to the grain

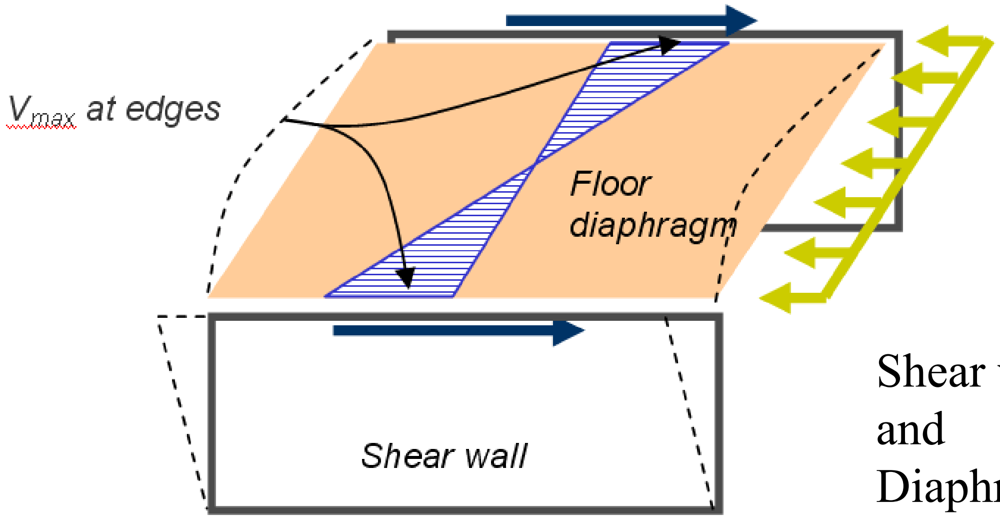


Where?



Truss

Wood I-joist



Shear wall and Diaphragm

Design principle

- Stress = Force / Area
- Tension resistance force = Tensile strength x Member cross sectional area

$$P_f \leq P_r$$

- Simplest member to design – no stability issue to be considered
- Member cross sectional area could be gross or net if there are holes in member.

Sawn lumber

$T_r = \phi F_t A_n K_{Zt}$ (Clause 6.5.9)

where

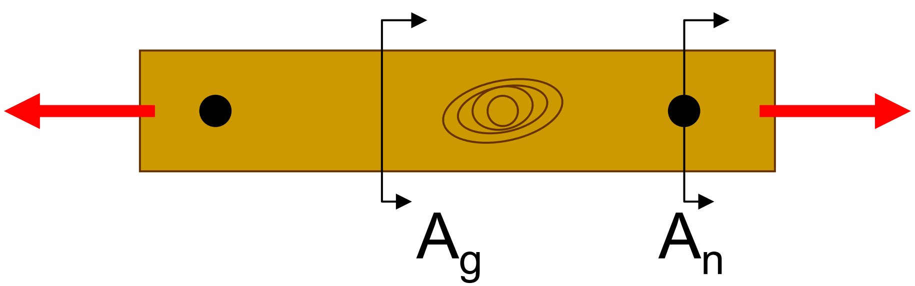
$\phi = 0.9$

$F_t = f_t (K_D K_H K_{St} K_T)$

f_t = specified tension strength parallel to grain

A_n = net area

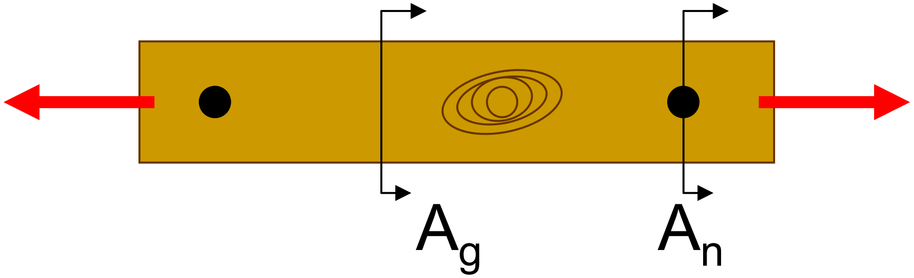
K_{Zt} = size factor (Table 6.4.5)



Larger dimension, mm	Bending and shear K_{Zb}, K_{Zv}			Tension parallel to grain, K_{Zt}
	Smaller dimension, mm			
	38 to 64	89 to 102	114 or more	All
38	1.7	—	—	1.5
64	1.7	—	—	1.5
89	1.7	1.7	—	1.5
114	1.5	1.6	1.3	1.4
140	1.4	1.5	1.3	1.3
184 to 191	1.2	1.3	1.3	1.2
235 to 241	1.1	1.2	1.2	1.1
286 to 292	1.0	1.1	1.1	1.0
337 to 343	0.9	1.0	1.0	0.9
387 or larger	0.8	0.9	0.9	0.8

Net area of cross section

- A_n - Gross area of cross section minus the projected area of all material removed by boring, grooving, notching etc.
- Fasteners that are considered to cause an area reduction: Split rings, shear plates, bolts, lag screws, and drift pins
- The area reduced due to bolts, lag screw or drift pin holes is equal to:
  the diameter of the hole (add 2 mm for bolts) x number of fasteners in the same critical section x the thickness of the member



$$A_n \geq 0.75\ A_g$$

Clause 5.3.8.2

Where

A_g = gross area of X-section in mm^2

Normally, fasteners will require removal of some wood.

Nails, timber rivets, truss plates do not require any material removal ($A_n = A_g$)

If $A_n < 0.75\ A_g$, change the fastener or increase A_g

Glulam

7.5.11 Tensile resistance parallel to grain

The factored tensile resistance parallel to grain, T_r, shall not be less than the maximum factored tensile force, T_f, and shall be calculated as the lesser of

$$T_r = \phi F_{tn} A_n$$

or

$$T_r = \phi F_{tg} A_g$$

where

ϕ = 0.9

$F_{tn} = f_{tn}(K_D K_H K_{St} K_T)$

where

f_{tn} = specified strength in tension parallel to grain at net section, MPa (Table 7.3)

A_n = net area of cross-section, mm^2

$F_{tg} = f_{tg}(K_D K_H K_{St} K_T)$

where

f_{tg} = specified strength in tension parallel to grain at gross section, MPa (Table 7.3)

A_g = gross area of cross-section, mm^2

About 75% of f_{tn}

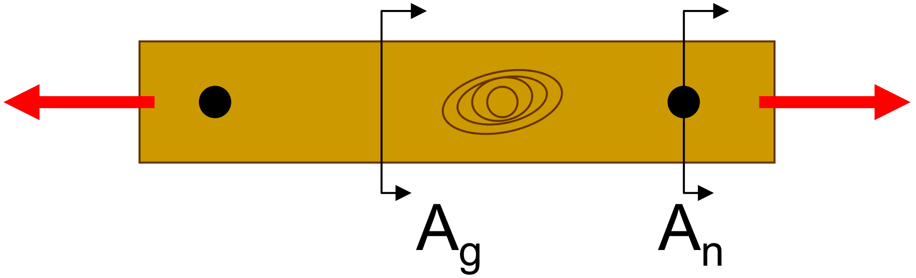


Table 7.3
Specified strengths and modulus of elasticity for glued-laminated timber, MPa

(See Clauses 7.5.9.3, 10.5.3, 10.5.4, 10.5.5, 10.6.3.1, 10.6.3.6, 10.6.3.7, A.6.5.6.3.6.)

	Douglas Fir-Larch					
	24f-E	24f-EX	20f-E	20f-EX	18t-E	16c-E
Bending moment (pos.), f_b	30.6	30.6	25.6	25.6	24.3	14.0
Bending moment (neg.), f_b	23.0	30.6	19.2	25.6	24.3	14.0
Longitudinal shear, f_v	2.0	2.0	2.0	2.0	2.0	2.0
Compression parallel, f_c	30.2*	30.2*	30.2*	30.2*	30.2	30.2
Compression parallel combined with bending, f_{cb}	30.2*	30.2	30.2*	30.2	30.2	30.2
Compression perpendicular, f_{cp} Compression face bearing	7.0	7.0	7.0	7.0	7.0	7.0
Tension face bearing	7.0	7.0	7.0	7.0	7.0	7.0
Tension net section, f_{tn} (see Clause 7.5.11)	20.4*	20.4	20.4*	20.4	23.0	20.4
Tension gross section, f_{tg}	15.3*	15.3	15.3*	15.3	17.9	15.3

DESIGN OF COMPRESSION MEMBERS

Compression member design

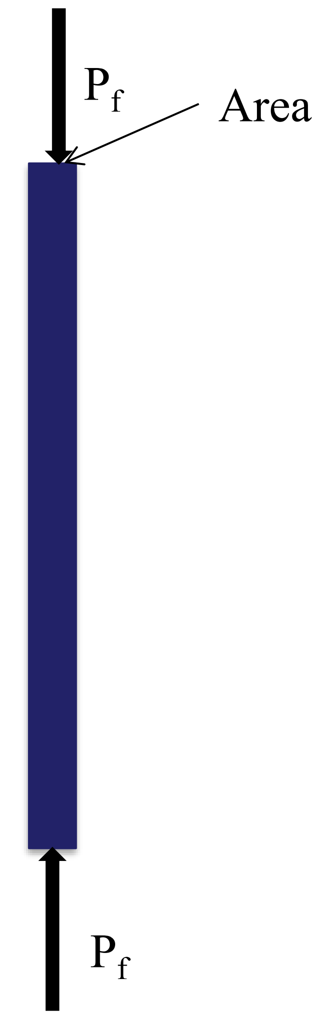
- Compressive resistance parallel to grain
  - Sawn lumber columns
  - Glulam columns
  - Stud walls
  - Trusses
- Compressive resistance perpendicular (bearing) or at an angle to grain

Design principle

$P_f \leq P_r$
P_f – worst case from load combinations

Force = Stress x Area

$P_r = \phi F_c A$



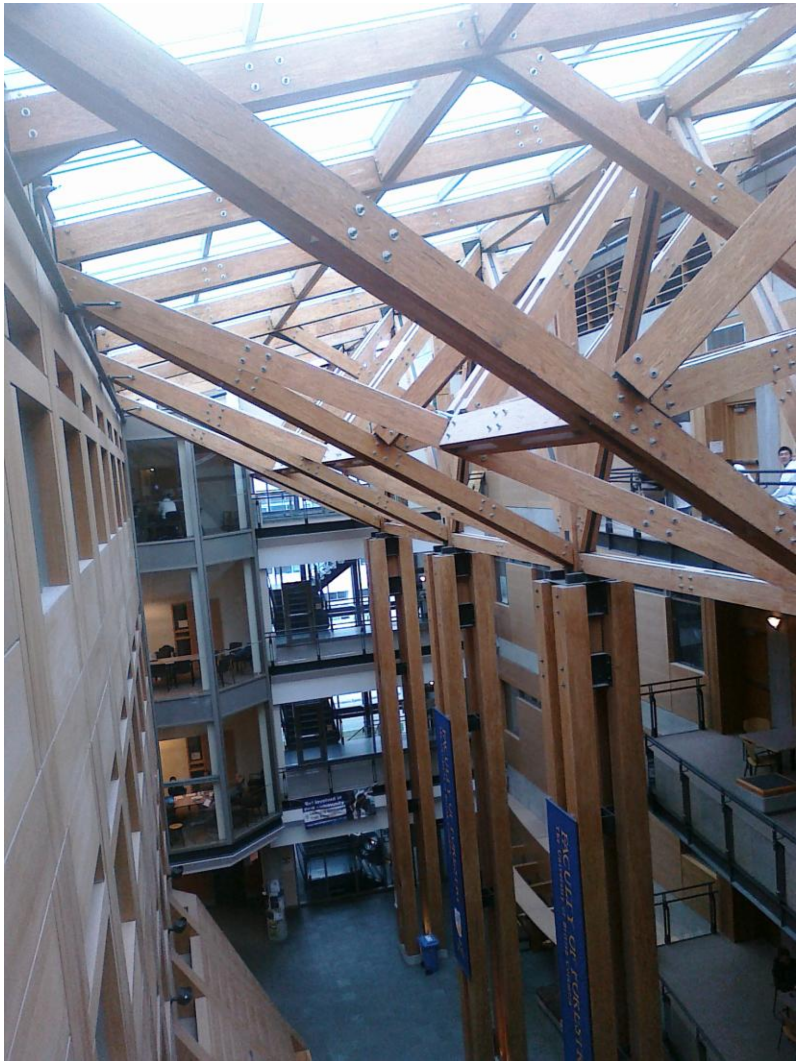
DESIGN OF COMPRESSION MEMBERS

Loaded parallel to grain

Column

Stud wall

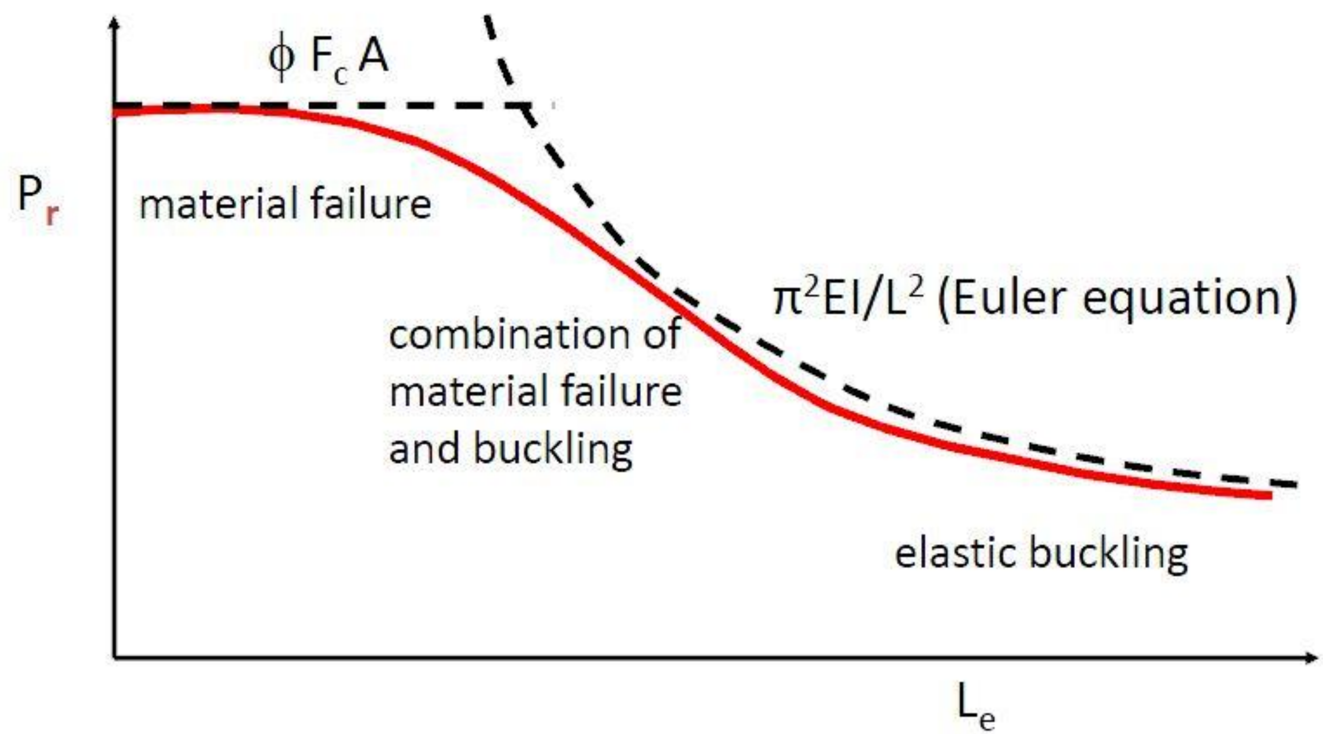
Truss

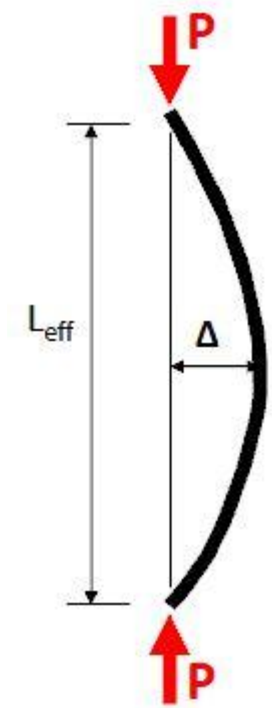


Column failure modes

- Short – wood crushing
  - Strength failure
- Long (slender) – buckling
  - Stability failure
- Intermediate – crushing & buckling

L_e – effective length (depends on end conditions)





Axially-loaded compression lumber members

6.5.6.2.3 Factored compressive resistance parallel to grain

The factored compressive resistance parallel to grain, P_r, shall be taken as follows:

$$P_r = \phi F_C A K_{Zc} K_C$$

where

ϕ = 0.8

$F_C = f_c (K_D K_H K_{Sc} K_T)$

where

f_c = specified strength in compression parallel to grain, MPa (Tables 6.3.1A to 6.3.1D, 6.3.2, and 6.3.3)

$K_{Zc} = 6.3\,(dL)^{-0.13} \leq 1.3$

where

d = dimension in direction of buckling (depth or width), mm

L = length associated with member dimension, mm

Need to check both axes!

Modification Factors

K_D – Load duration
K_H – System
K_{Sc} – Service condition for compression
K_T – Chemical treatment

$K_{Zcd} = 6.3(dL)^{-0.13} \leq 1.3$ or $K_{Zcb} = 6.3(bL)^{-0.13} \leq 1.3$

d, b = dimension in direction of buckling (depth or width) (mm)

6.5.6.2.4 Slenderness factor, K_C

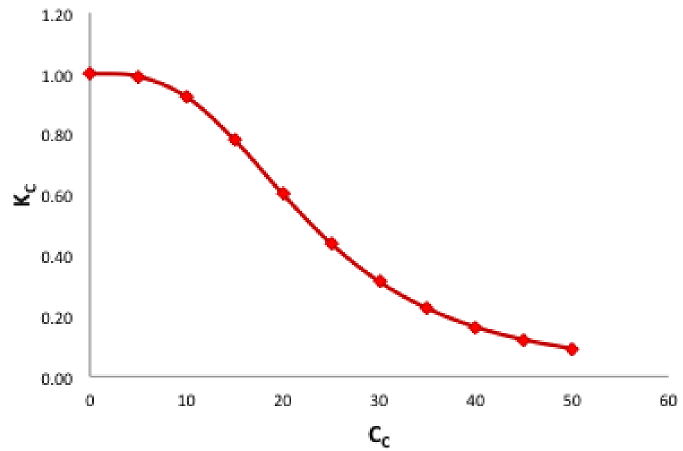
For both axes, the slenderness factor, K_C, shall be taken as follows:

$$K_C = \left[1.0 + \frac{F_C K_{Zc} C_c^3}{35 E_{05} K_{SE} K_T}\right]^{-1}$$

K_{SE} – service condition for E

where

E_{05} = 0.82E for MSR lumber
= 0.75E for MEL lumber
= as specified in Tables 6.3.1A to 6.3.1D for visually graded lumber



Slenderness ratio, C_c

6.5.6.2.2 Constant rectangular cross-section

The slenderness ratio, C_C, of simple compression members of constant rectangular section shall not exceed 50 and shall be calculated for both axes as follows:

$$C_C = \frac{\text{effective length associated with width}}{\text{member width}}$$

$$C_C = \frac{\text{effective length associated with depth}}{\text{member depth}}$$

where K_eL= Effective length

K_e- Effective length factor

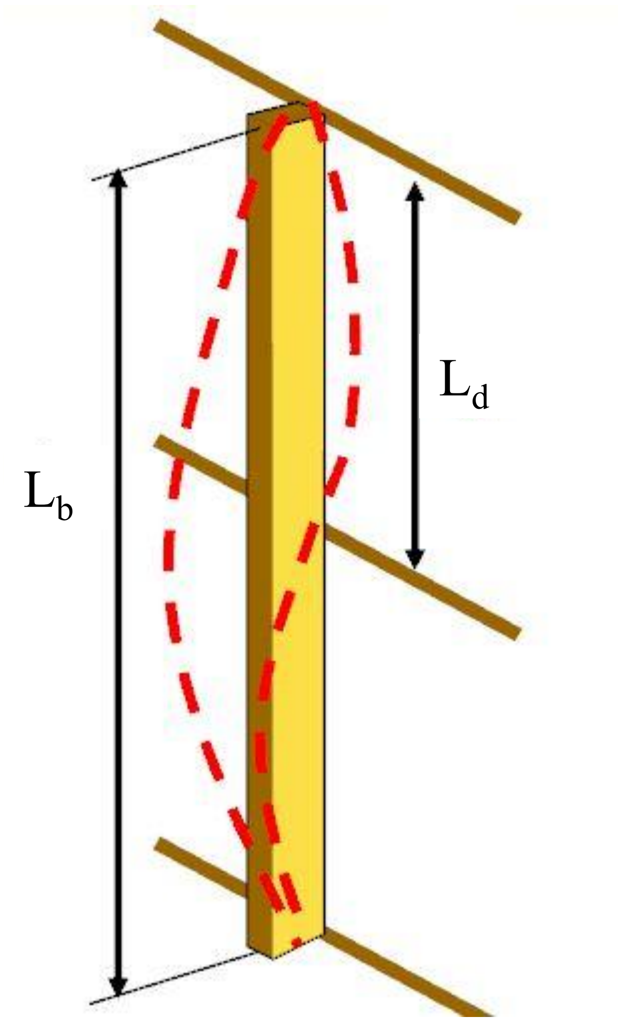
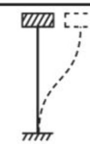
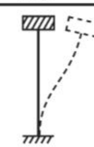
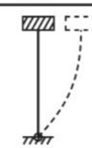


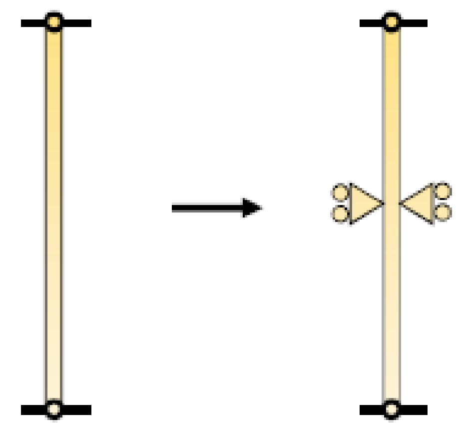
Table A.6.5.6.1
Minimum design values of effective length factor, K_e, for compression members

Degree of end restraint of compression member	Effective length factor, K_e	Symbol
Effectively held in position and restrained against rotation at both ends	0.65	
Effectively held in position at both ends and restrained against rotation at one end	0.80	
Effectively held in position at both ends but not restrained against rotation	1.00	
Effectively held in position and restrained against rotation at one end, and at the other end restrained against rotation but not held in position	1.20	
Effectively held in position and restrained against rotation at one end, and at the other partially restrained against rotation but not held in position	1.50	
Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position	2.00	
Effectively held in position and restrained against rotation at one end but not held in position or restrained against rotation at the other end	2.00	

Slenderness ratio, C_c

If > 50, the designer has two options:

- add lateral supports



-increase size

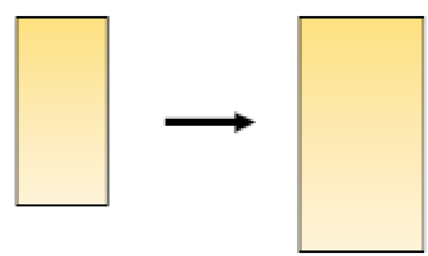


Table 6.3.1A
Specified strengths and modulus of elasticity for structural joist and plank, structural light framing, and stud grade categories of lumber, MPa

Species identification	Grade	Bending at extreme fibre, f_b	Longitudinal shear, f_v	Compression		Tension parallel to grain, f_t	Modulus of elasticity	
				Parallel to grain, f_c	Perpendicular to grain, f_{cp}		E	E_{05}
D Fir-L	SS	16.5		19.0		10.6	12 500	8 500
	No. 1/No. 2	10.0	1.9	14.0	7.0	5.8	11 000	7 000
	No. 3/Stud	4.6		7.3		2.1	10 000	5 500
Hem-Fir	SS	16.0		17.6		9.7	12 000	8 500
	No. 1/No. 2	11.0	1.6	14.8	4.6	6.2	11 000	7 500
	No. 3/Stud	7.0		9.2		3.2	10 000	6 000
Spruce-Pine-Fir	SS	16.5		14.5		8.6	10 500	7 500
	No. 1/No. 2	11.8	1.5	11.5	5.3	5.5	9 500	6 500
	No. 3/Stud	7.0		9.0		3.2	9 000	5 500
Northern	SS	10.6		13.0		6.2	7 500	5 500
	No. 1/No. 2	7.6	1.3	10.4	3.5	4.0	7 000	5 000
	No. 3/Stud	4.5		5.2		2.0	6 500	4 000

Note: *Tabulated values are based on the following standard conditions:*
(a) 286 mm *larger dimension;*
(b) dry service conditions; and
(c) standard-term duration of load.

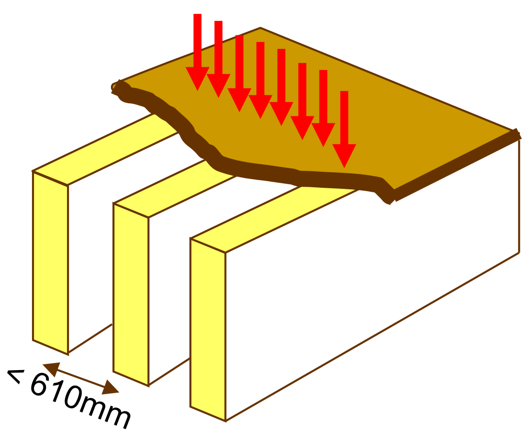
Table 6.4.2
Service condition factors, K_S

K_S	Property	Dry service conditions	Wet service conditions: sawn lumber, piling, and poles of least dimension	
			89 mm or less	Over 89 mm
K_{Sb}	Bending at extreme fibre	1.00	0.84	1.00
K_{Sf}	Fracture shear	1.00	0.70	0.70
K_{Sv}	Longitudinal shear	1.00	0.96	1.00
K_{Sc}	Compression parallel to grain	1.00	0.69	0.91
K_{Scp}	Compression perpendicular to grain	1.00	0.67	0.67
K_{St}	Tension parallel to grain	1.00	0.84	1.00
K_{SE}	Modulus of elasticity	1.00	0.94	1.00

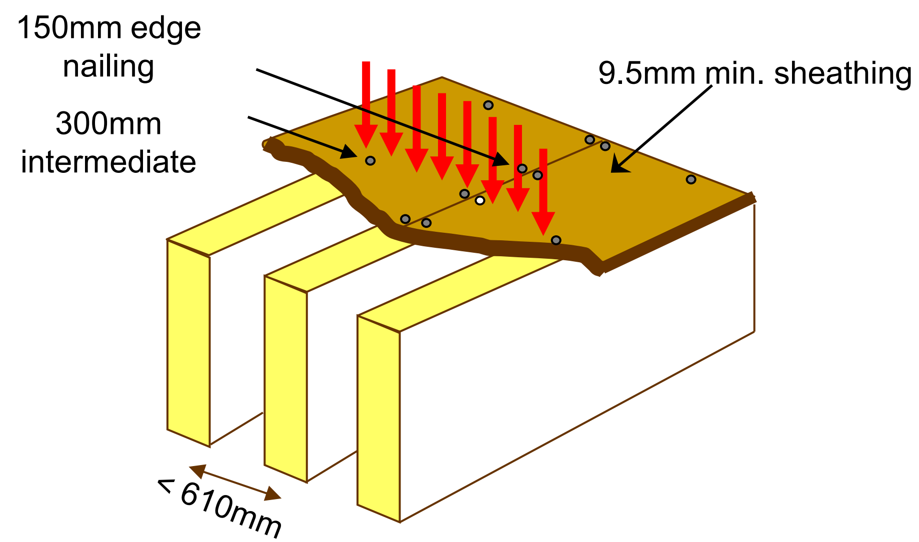
Table 6.4.4
System factor, K_H

For specified strength in	Case 1*	Case 2† Visually graded	Case 2† MSR	Built-up beams
Bending	1.10	1.40	1.20	1.10
Longitudinal shear	1.10	1.40	1.20	1.10
Compression parallel to grain	1.10	1.10	1.10	1.00
Tension parallel to grain	1.10	—	—	1.00
All other properties	1.00	1.00	1.00	1.00

*See Clause 6.4.4.1 for conditions applying to Case 1.
†See Clause 6.4.4.2 for conditions applying to Case 2.



Case 1

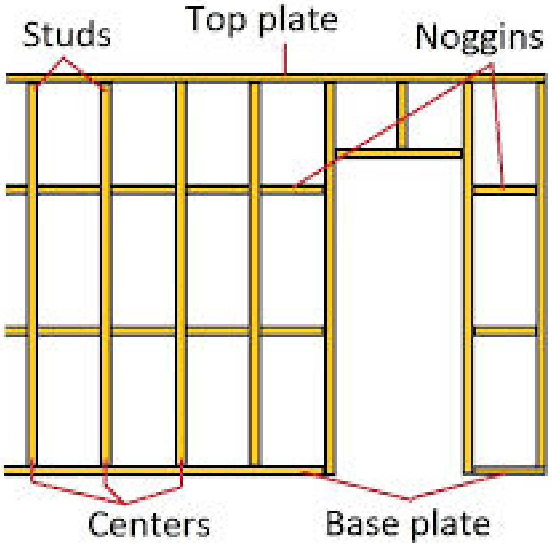


Case 2

6.5.6.5 Stud walls

When stud walls are adequately sheathed on at least one side, as in light frame construction, the dimension of the stud normal to the sheathing may be used in calculating the slenderness ratio.

- Wall sheathing and blocking provides support against buckling about the weak axis of the stud
- Permissible to calculate P_r in stronger axis and ignore the weaker axis



Axially-loaded compression glulam members

7.5.8.4 Factored compressive resistance parallel to grain

7.5.8.4.1

Bending moments due to eccentrically applied axial loads shall be taken into account in accordance with Clause 7.5.12.

7.5.8.4.2

The factored compressive resistance parallel to grain, P_r, shall be taken as follows:

$$P_r = \phi F_c A K_{Zcg} K_C$$

where

ϕ = 0.8

$F_c = f_c(K_D K_H K_{Sc} K_T)$

where

f_c = specified strength in compression parallel to grain, MPa (Table 7.3)

$K_{Zcg} = 0.68(Z)^{-0.13} \leq 1.0$

where

Z = member volume, m^3

Volume effect in glulam

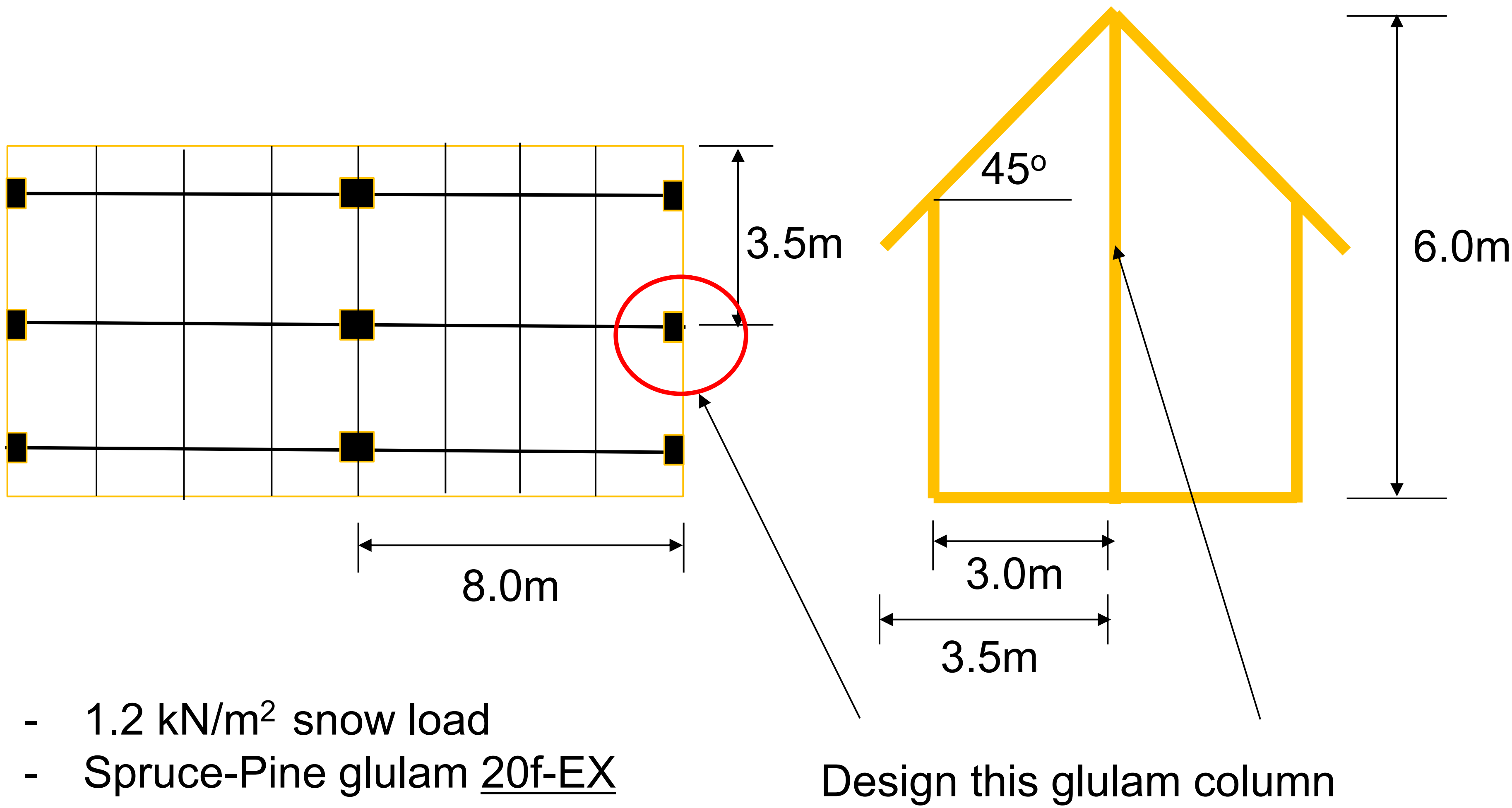
K_C = slenderness factor (Clause 7.5.8.5)

Recap - Design Process

- Determine effective length
- Select member size, grade, species group
- Calculate factored resistance parallel to grain
- Select another size and recalculate if needed ($P_r < P_f$)

Exercise

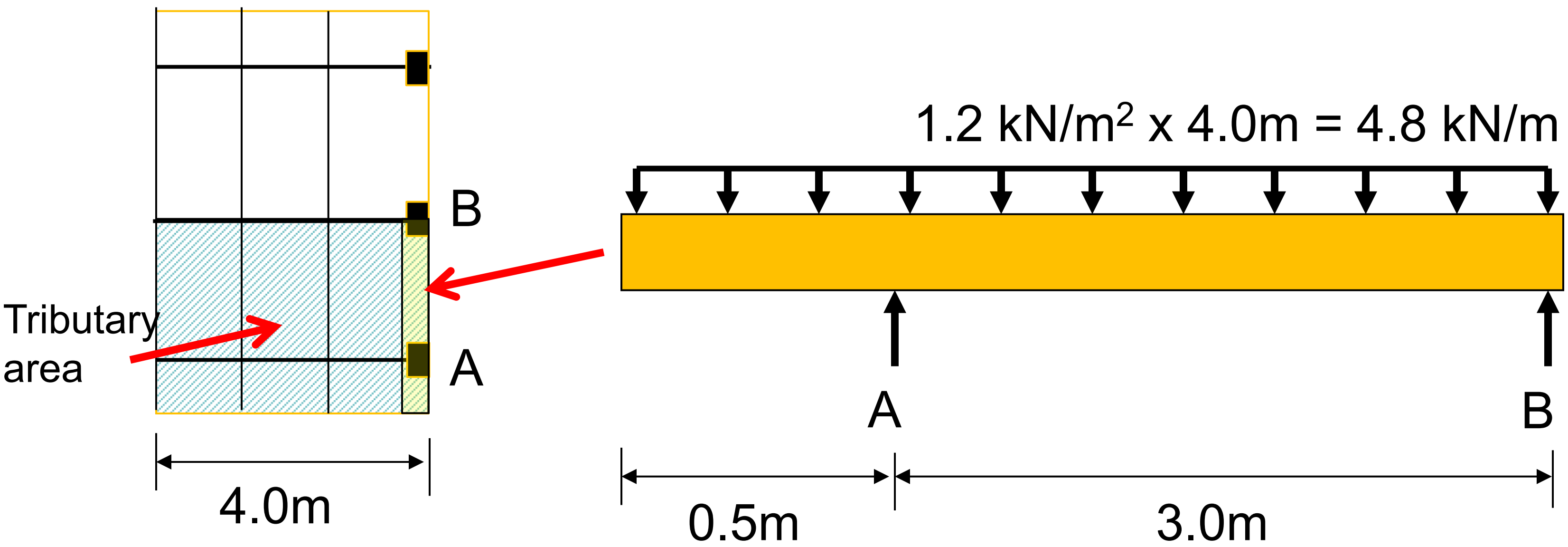
Glulam Column



- 1.2 kN/m² snow load
- Spruce-Pine glulam 20f-EX
- $K_D = K_{SC} = K_T = K_H = 1$

Determine the min. dimensions of the X- section

Determine the Factored Force on the Post Assuming Pin Connections



Sum of M_A = 4.8(0.5)(0.25) + 4.8(3.0)(1.5) – B(3.0) = 0

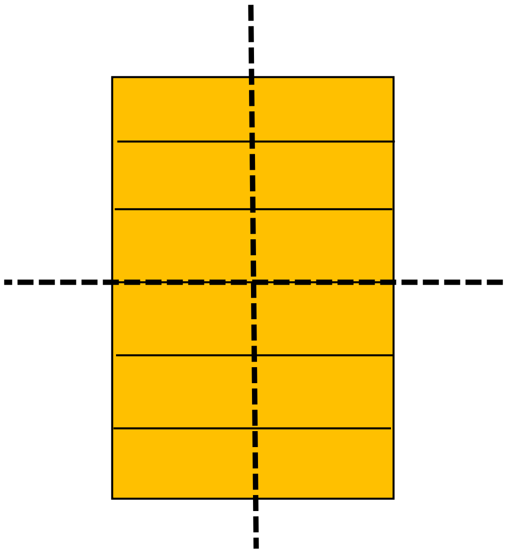
→ B = 7.0kN

Axial load on post = 7.0 x 2 (sides) = 14.0kN

Load case without wind and dead load

→ **P_f = 1.5 (14.0kN) = 21.0 kN**

Determine Min. Section for Slenderness



$50 = L_e/d = 6000/d$ (50 is max.)

$d = 6000/50 = 120mm$ (min.)

Remember: C_c = Maximum $(C_{C1}, C_{C2}) \leq 50$

Use 130 x 152 mm as a trial section

- Spruce-Pine glulam <u>20f-EX</u>

- $K_D = K_{SC} = K_T = K_H = 1$

$$\left.\begin{aligned} C_c &= \frac{L_{ew}}{W} = \frac{6000}{130} = 46.1 \\ C_c &= \frac{L_{ed}}{d} = \frac{6000}{152} = 39.4 \end{aligned}\right\} \leq 50$$

Therefore $C_c = 46.1$

Table 7.3
Specified strengths and modulus of elasticity for glued-laminated timber, MPa

	Spruce-Lodgepole Pine-Jack Pine			
	20f-E	20f-EX	14t-E	12c-E
Bending moment (pos.), f_b	25.6	25.6	24.3	9.8
Bending moment (neg.), f_b	19.2	25.6	24.3	9.8
Longitudinal shear, f_v	1.75	1.75	1.75	1.75
Compression parallel, f_c	25.2*	25.2*	25.2	25.2
Compression parallel combined with bending, f_{cb}	25.2*	25.2	25.2	25.2
Compression perpendicular, f_{cp} Compression face bearing	5.8	5.8	5.8	5.8
Tension face bearing	5.8	5.8	5.8	5.8
Tension net section, f_{tn} (see Clause 7.5.11)	17.0*	17.0	17.9	17.0
Tension gross section, f_{tg}	12.7*	12.7	13.4	12.7
Tension perpendicular to grain, f_{tp}	0.51	0.51	0.51	0.51
Modulus of elasticity, E	10 300	10 300	10 700	9 700

**The use of this stress grade for this primary application is not recommended.*

$E_{05} = 0.87E$

$$P_r = \phi F_c A K_c K_{zcg}$$

For 20f-EX glulam,

$f_C = 25.2$ MPa	(Table 7.3)
$E = 10300$ MPa	

$$F_C = 25.2\,(1 \text{x} 1 \text{x} 1 \text{x} 1) = 25.2 \text{ MPa}$$

$$K_{zcg} = 0.68\,(Z)^{-0.13}$$

$$= 0.68(0.13 \times .152 \times 6)^{-0.13}$$

$$= 0.897$$

Remember: $K_{zcg} = 0.68\,(Z)^{-0.13} \leq 1.0$ *Volume factor*

$$\mathbf{P_r = \phi\, F_c\, A\, K_c\, K_{zcg}}$$

$F_C = 25.2\ (1x1x1x1) = 25.2$ MPa

$K_{zcg} = 0.68\ (Z)^{-0.13} = 0.68\ (0.13 \times .152 \times 6)^{-0.13} = 0.897$

$$K_c = \left[1.0 + \frac{F_c\, K_{Zcg}\, C_c^{3}}{35\, E_{05}\, K_{SE}\, K_T} \right]^{-1}$$

$$K_c = \left[1.0 + \frac{25.2\ (0.897)\ (46.1)^{3}}{35\ (0.87x10300)x1x1} \right]^{-1}$$

$$\mathbf{K_c = 0.12}$$

$$P_r = \phi F_c A K_c K_{zcg}$$

$$P_r = 0.8\ (25.2)\ (130 \times 152)\ (0.897)\ (0.12)$$
$$= 42\ 879\ \text{N}$$

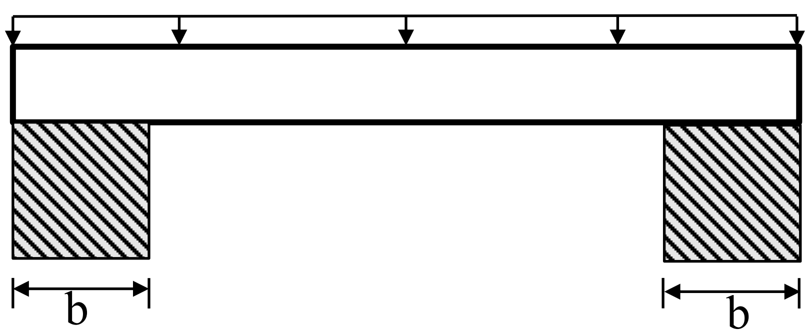
$$P_r = 42.9\ \text{kN} \quad > \quad 21\ \text{kN} \quad \text{O.K.}$$

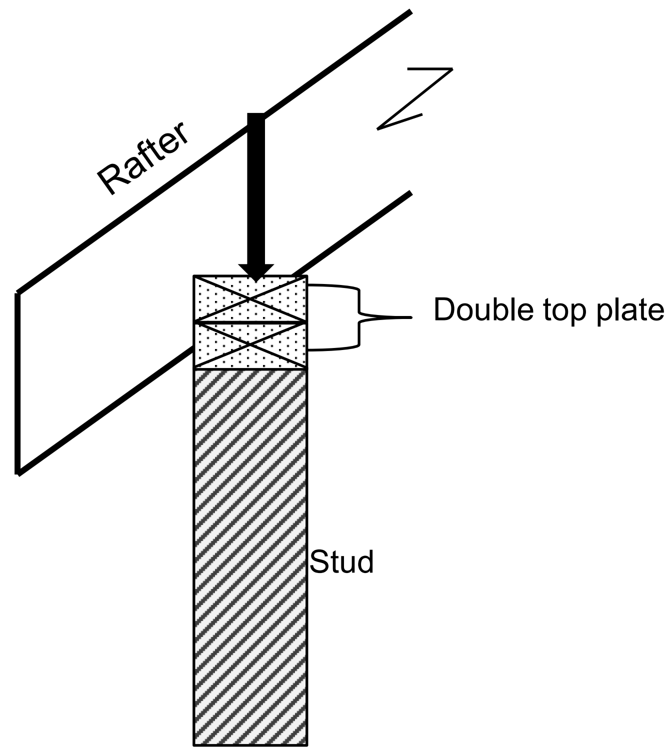
<u>Use 130 x 152 mm</u>

DESIGN OF COMPRESSION MEMBERS

Loaded perpendicular to grain

Compression perpendicular to grain check – bearing at supports and load points





Compression perpendicular-to-grain (bearing) resistance of lumber

Identical provisions for glulam

6.5.7 Compressive resistance perpendicular to grain

Note: *See Clause A.6.5.7.*

6.5.7.2 Effect of all applied loads

The factored compressive resistance perpendicular to grain under the effect of all factored applied loads shall be taken as Q_r as follows:

$$Q_r = \phi F_{cp} A_b K_B K_{Zcp}$$

where

ϕ = 0.8

$F_{cp} = f_{cp}(K_D K_{Scp} K_T)$

where

f_{cp} = specified strength in compression perpendicular to grain, MPa (Tables 6.3.1A to 6.3.1D, 6.3.2, and 6.3.3)

A_b = bearing area, mm^2

K_B = length of bearing factor (Clause 6.5.7.5)

K_{Zcp} = size factor for bearing (Clause 6.5.7.4)

Compression perpendicular-to-grain (bearing) resistance of lumber

Identical provisions for glulam

6.5.7 Compressive resistance perpendicular to grain

Note: *See Clause A.6.5.7.*

6.5.7.3 Effect of loads applied near a support

6.5.7.3.1 Factored compressive resistance perpendicular to grain

The factored compressive resistance perpendicular to grain under the effect of only those loads applied within a distance from the centre of the support equal to the depth of the member shall be taken as Q'_r as follows:

$$Q'_r = (2/3)\ \phi F_{cp} A'_b K_B K_{Zcp}$$

where

$\phi = 0.8$

$F_{cp} = f_{cp}(K_D K_{Scp} K_T)$

A'_b = average bearing area, mm^2 (see Clause 6.5.7.3.2)

$$A'_b = b\left(\frac{L_{b1} + L_{b2}}{2}\right), \leq 1.5b(L_{b1})$$

where

b = average bearing width (perpendicular to grain), mm

L_{b1} = lesser bearing length, mm

L_{b2} = larger bearing length, mm

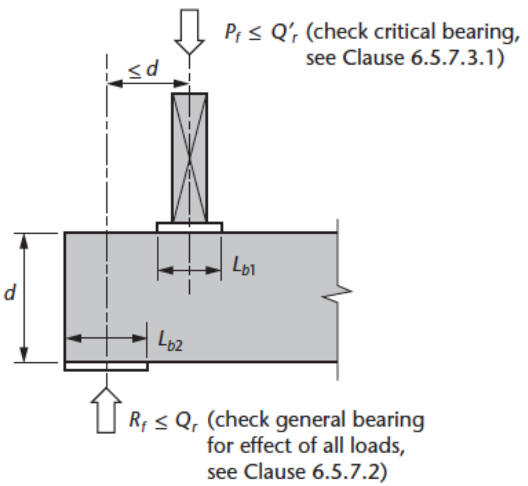


Figure 6.5.7.3
Load applied near a support

Modification factors

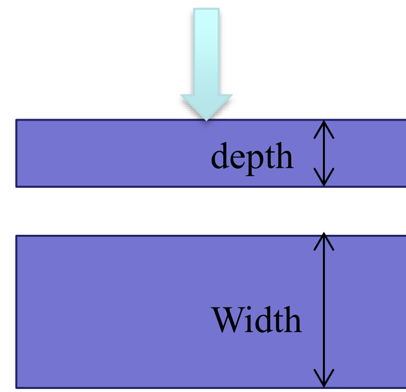
6.5.7.4 Size factor for bearing, K_{Zcp}

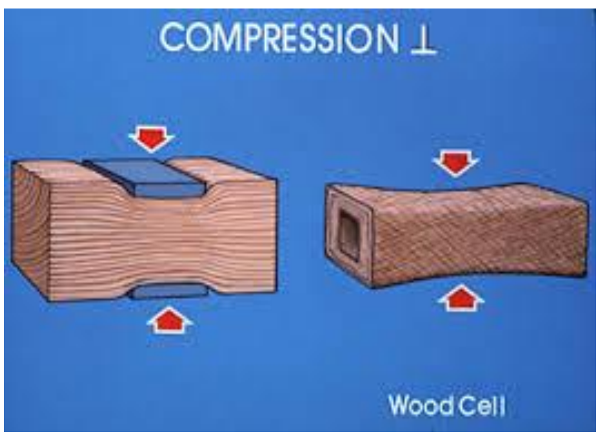
When the width of a member (dimension perpendicular to the direction of the load) is greater than the depth of the member (dimension parallel to the direction of the load), the specified strength in compression perpendicular to grain may be multiplied by a size factor for bearing, K_{Zcp}, in accordance with Table 6.5.7.4.

Table 6.5.7.4
Size factor for bearing, K_{Zcp}

Ratio of member width to member depth*	K_{Zcp}
1.0 or less	1.00
2.0 or more	1.15

**Interpolation applies for intermediate ratios.*





Test method – strength depends on bearing area

Modification factors

6.5.7.5 Length of bearing factor, K_B

When lengths of bearing or diameters of washers are less than 150 mm, specified strengths in compression perpendicular to grain may be multiplied by a length of bearing factor, K_B, in accordance

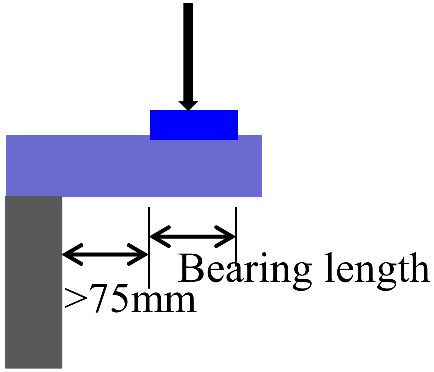
with Table 6.5.7.5, provided that

(a) no part of the bearing area is less than 75 mm from the end of the members; and

(b) bearing areas do not occur in positions of high bending stresses.

Table 6.5.7.5
Length of bearing factor, K_B

Bearing length (parallel to grain) or washer diameter, mm	Modification factor, K_B
12.5 and less	1.75
25.0	1.38
38.0	1.25
50.0	1.19
75.0	1.13
100.0	1.10
150.0 or more	1.00



Compression at an angle

6.5.8 Compressive resistance at an angle to grain

The factored compressive resistance at an angle to grain shall be taken as follows:

$$N_r = \frac{P_r Q_r}{P_r \sin^2\theta + Q_r \cos^2\theta}$$

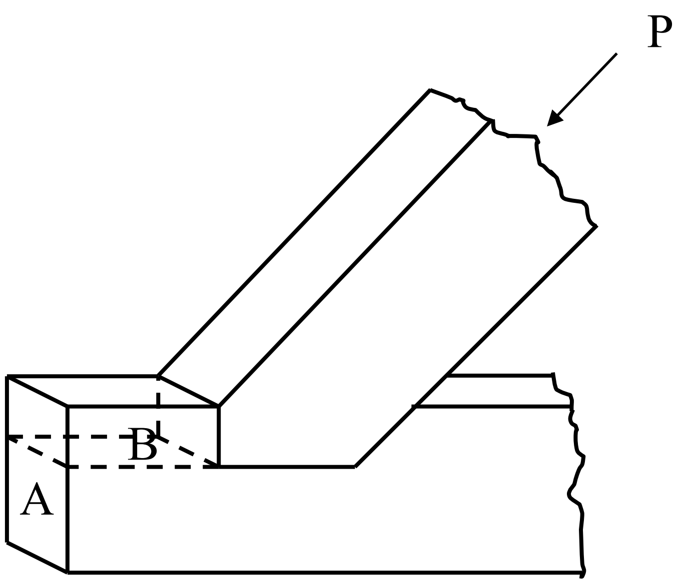
Hankinson's formula

where

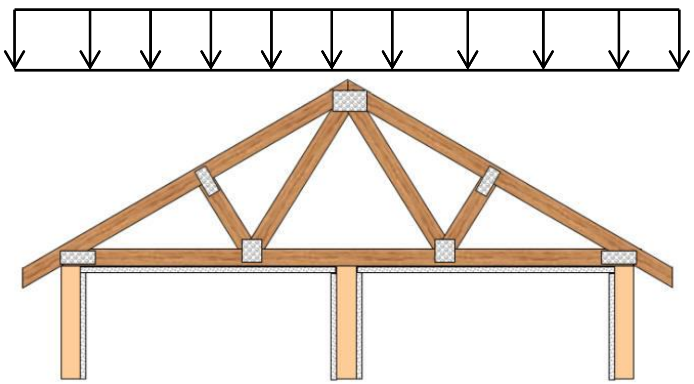
P_r = factored compressive resistance parallel to grain, N (Clause 6.5.6.2.3, assuming $K_C = 1.00$)

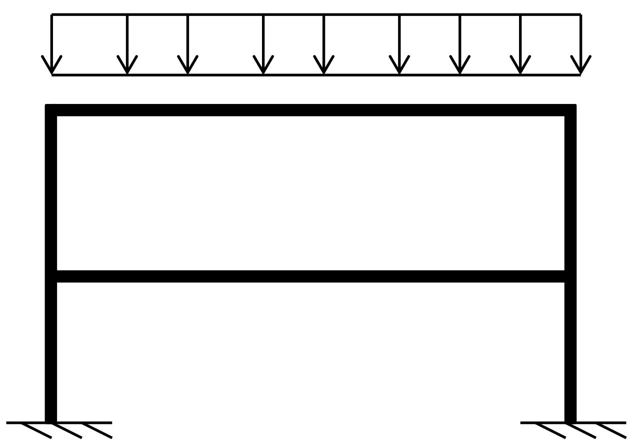
Q_r = factored compressive resistance perpendicular to grain, N (Clause 6.5.7.2)

θ = angle between direction of grain and direction of load, degrees



Beam-columns

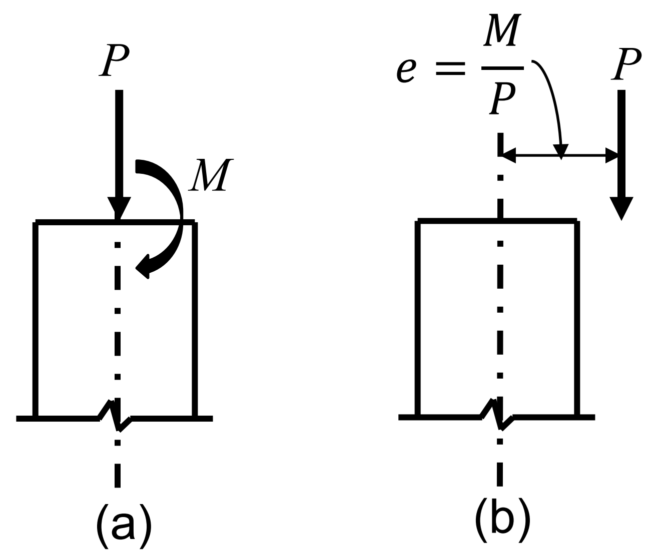




Equivalent eccentricity of a column load

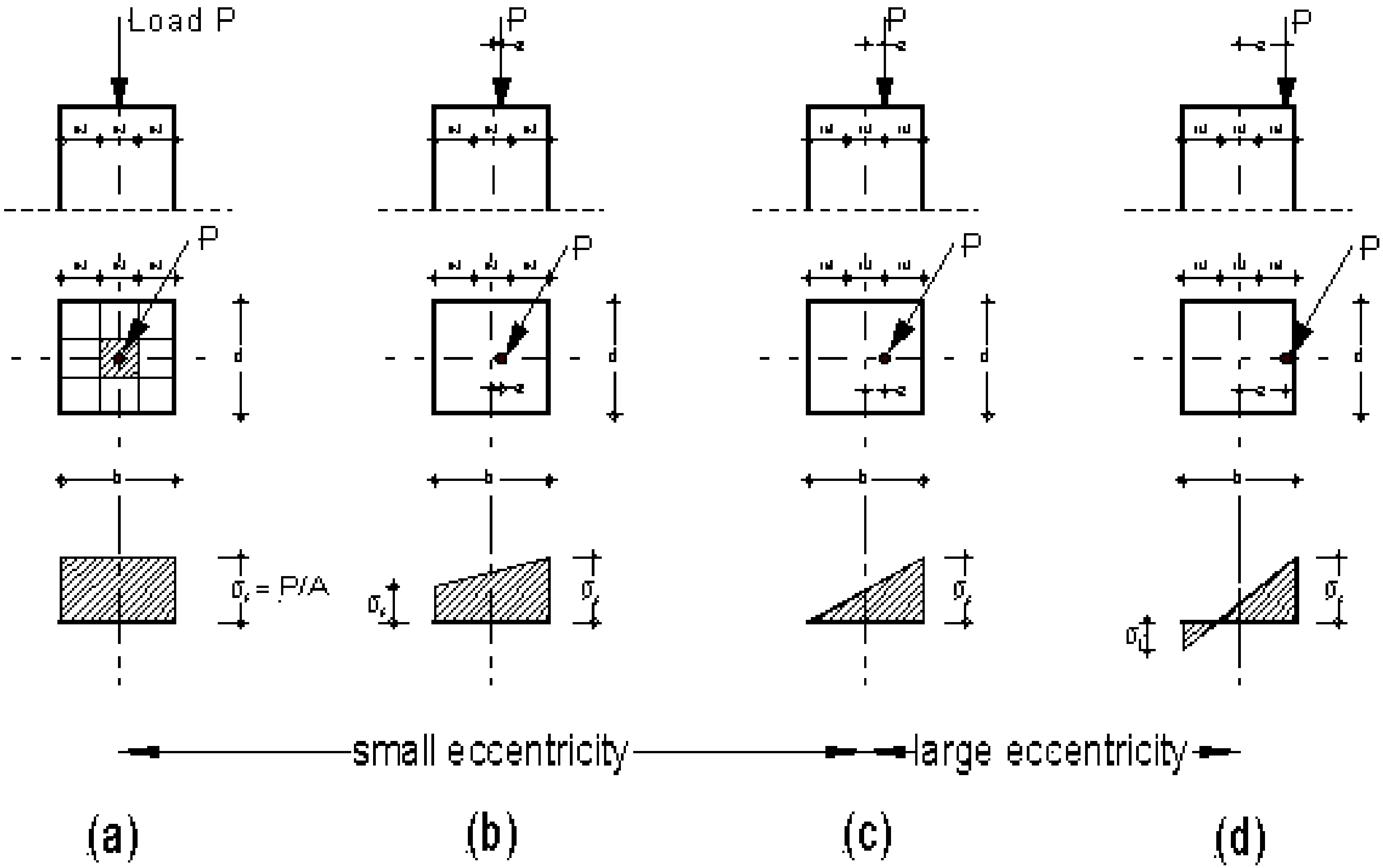
(a) Concentrically loaded column

(b) Eccentrically loaded column



Column – eccentricity is often assumed in design even in axially loaded column to account for fabrication imperfection and construction tolerance.

Stress distribution in a beam-column



Note : Eccentricity can be present in two orthogonal directions

Combined bending and axial force – Sawn lumber and glulam

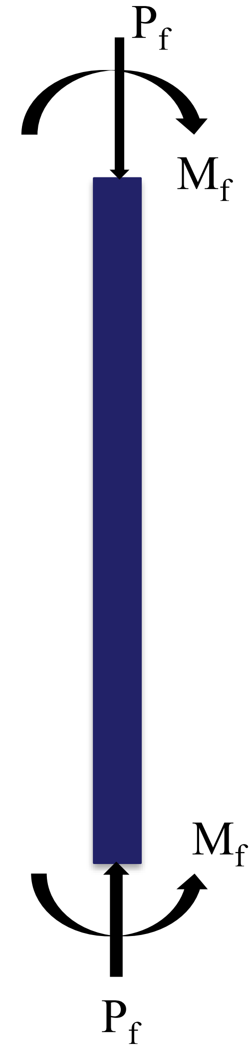
6.5.10 Resistance to combined bending and axial load

Members subject to combined bending and compressive or tensile axial loads shall be designed to satisfy the appropriate interaction equation:

$$\left(\frac{P_f}{P_r}\right)^2 + \frac{M_f}{M_r}\left[\frac{1}{1-\frac{P_f}{P_E}}\right] \le 1$$

or

$$\frac{T_f}{T_r} + \frac{M_f}{M_r} \le 1$$



Subscript : r – factored resistance ; f – factored load effect

P_E = Euler buckling load in the plane of the applied moment

$$= \frac{\pi^2 E_{05} K_{SE} K_T I}{L_e^2}$$

where

E_{05} = modulus of elasticity for design of compression members, MPa

I = moment of inertia in the plane of the applied moment, mm^4

L_e = effective length in the plane of the applied moment

$= K_e L$

where

K_e = the effective length factor given in Clause A.6.5.6.1

Slide from bending member design lecture

Sawn lumber bending members

6.5.4 Bending moment resistance

6.5.4.1 General

The factored bending moment resistance, M_r, of sawn lumber members shall be taken as follows:

$$M_r = \phi F_b S K_{Zb} K_L$$

where

ϕ = 0.9

F_b = $f_b(K_D K_H K_{Sb} K_T)$

where

f_b = specified strength in bending, MPa (Tables 6.3.1A to 6.3.1D, 6.3.2, and 6.3.3)

K_{Zb} = size factor in bending (Clause 6.4.5)

K_L = lateral stability factor (Clause 6.5.4.2)

The resistance may be governed by material strength or lateral stability ($K_L < 1$) .

S = Section modulus, mm^3

End
Lecture #3

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