Lecture #4 – Design of bending members

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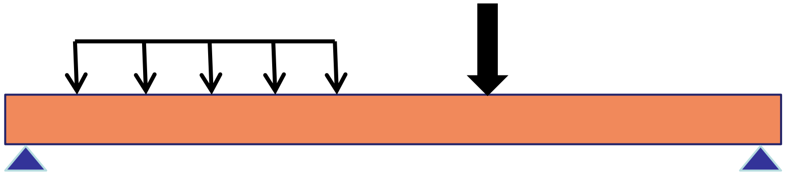
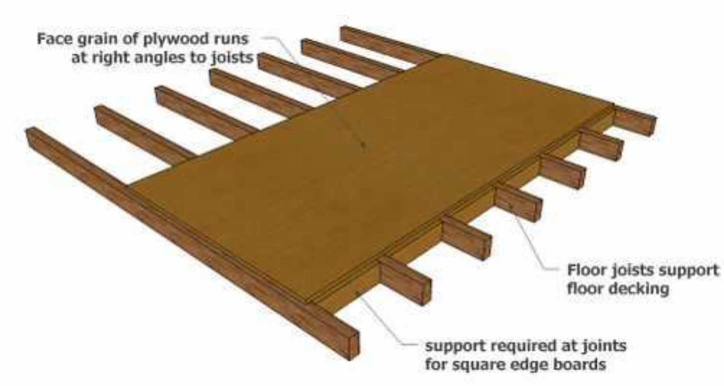
Design of bending members

- Sawn lumber bending members
- Glulam bending members
- Composite beam with semi-rigid connection
- CLT panel design

General information

- Bending members
  - subjected to out-of-plane loading applied perpendicular to the longitudinal axis of beam or plane of panel
- Applications
  - Joists, beams and purlins
  - Sheathing and decking





Potential limit states

- Design must consider 6 limit states
  - Bending moment (extreme fibre stress)
    - generally governs for average loads and spans
  - Lateral stability of compression edge
  - Longitudinal shear
    - generally governs for heavy loads and short spans
  - Bearing at support and load points
  - Deflection
    - generally governs for light loads and long spans
  - Vibration
    - causing human discomfort

One step

Sawn lumber –
dimension lumber (2” wide) and timber (wider than 2”)

Sawn lumber categories

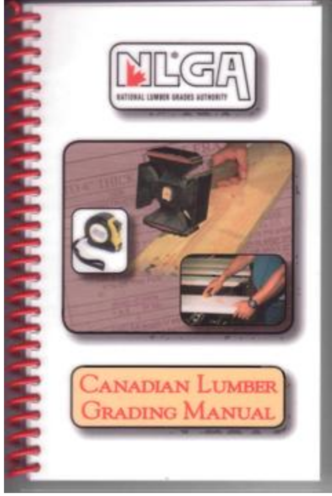


Table 6.2.2.1
Visual grades and their dimensions

Grade category	Smaller dimension, mm	Larger dimension, mm	Grades
Light framing	38 to 89	38 to 89	Construction, Standard
Stud	38 to 89	38 or more	Stud
Structural light framing	38 to 89	38 to 89	Select Structural No. 1, No. 2, No. 3
Structural joists and planks	38 to 89	114 or more	Select Structural No. 1, No. 2, No. 3
Beam and stringer	114 or more	Exceeds smaller dimension by more than 51	Select Structural No. 1, No. 2
Post and timber	114 or more	Exceeds smaller dimension by 51 or less	Select Structural No. 1, No. 2
Plank decking	38 to 89	140 or more	Select, Commercial

Species groups and grades

Two items dictate design properties of a piece of lumber:

- Species group to which the piece belong
- Grade (visual or machine grade)

Table 6.2.1.2
Species combinations

Species combinations	Stamp identification	Species included in the combination
Douglas Fir-Larch	D Fir-L (N)	Douglas fir, western larch
Hem-Fir	Hem-Fir (N)	Pacific coast hemlock, amabilis fir
Spruce-Pine-Fir	S-P-F	Spruce (all species except coast Sitka spruce), Jack pine, lodgepole pine, balsam fir, alpine fir
Northern Species	North Species	Any Canadian species graded in accordance with the NLGA rules

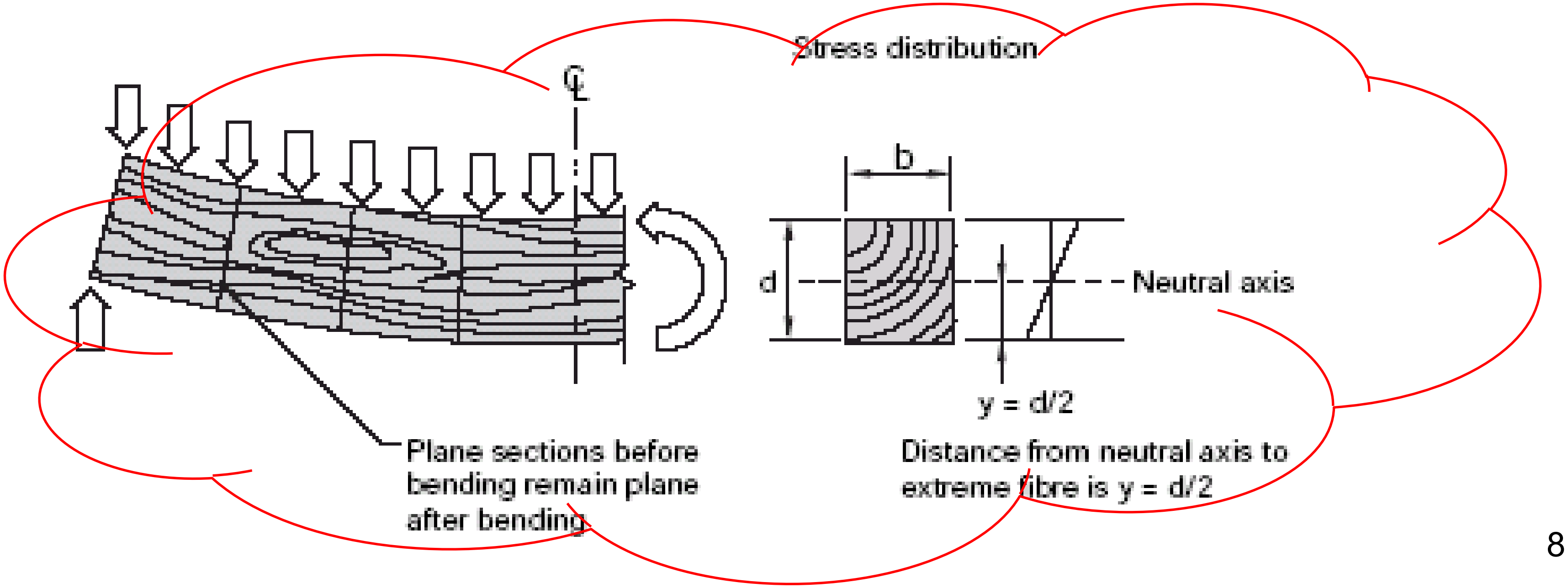
Bending moment capacity

Engineering Beam Equation,

$$\sigma_{max} = \frac{M \cdot y_{max}}{I} = \frac{M}{S}$$

$$M = \sigma_{max} S$$

where S = section modulus = I / y_{max}



Sawn lumber bending members

6.5.4 Bending moment resistance

6.5.4.1 General

The factored bending moment resistance, M_r, of sawn lumber members shall be taken as follows:

$$M_r = \phi F_b S K_{Zb} K_L$$

where

ϕ = 0.9

F_b = $f_b(K_D K_H K_{Sb} K_T)$

where

f_b = specified strength in bending, MPa (Tables 6.3.1A to 6.3.1D, 6.3.2, and 6.3.3)

K_{Zb} = size factor in bending (Clause 6.4.5)

K_L = lateral stability factor (Clause 6.5.4.2)

S = Section modulus, mm^3

The resistance may be governed by material strength or lateral stability ($K_L < 1$) .

Table 6.4.5
Size factor, K_Z, for visually stress-graded lumber

Larger dimension, mm	Bending and shear K_{Zb}, K_{Zv}			Tension parallel to grain, K_{Zt}	Compression perpendicular to grain, K_{Zcp}	Compression parallel to grain, K_{Zc}	All other properties
	Smaller dimension, mm						
	38 to 64	89 to 102	114 or more	All	All	All	All
38	1.7	—	—	1.5	See Clause 6.5.7.5	Value computed using formula in Clause 6.5.6.2.3	1.0
64	1.7	—	—	1.5			1.0
89	1.7	1.7	—	1.5			1.0
114	1.5	1.6	1.3	1.4			1.0
140	1.4	1.5	1.3	1.3			1.0
184 to 191	1.2	1.3	1.3	1.2			1.0
235 to 241	1.1	1.2	1.2	1.1			1.0
286 to 292	1.0	1.1	1.1	1.0			1.0
337 to 343	0.9	1.0	1.0	0.9			1.0
387 or larger	0.8	0.9	0.9	0.8			1.0

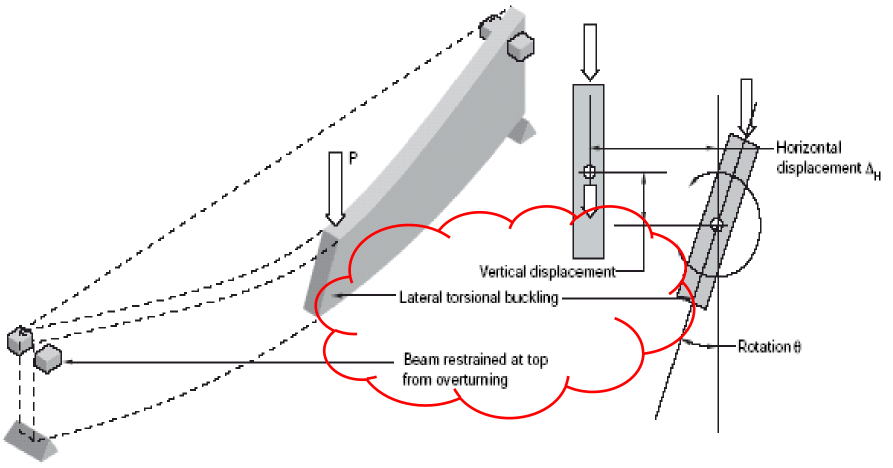
Table 6.3.1A
Specified strengths and modulus of elasticity for structural joist and plank, structural light framing, and stud grade categories of lumber, MPa

Species identification	Grade	Bending at extreme fibre, f_b	Longi-tudinal shear, f_v	Compression: Parallel to grain, f_c	Compression: Perpen-dicular to grain, f_{cp}	Tension parallel to grain, f_t	Modulus of elasticity: E	Modulus of elasticity: E_{05}
D Fir-L	SS	16.5		19.0		10.6	12 500	8 500
	No. 1/No. 2	10.0	1.9	14.0	7.0	5.8	11 000	7 000
	No. 3/Stud	4.6		7.3		2.1	10 000	5 500
Hem-Fir	SS	16.0		17.6		9.7	12 000	8 500
	No. 1/No. 2	11.0	1.6	14.8	4.6	6.2	11 000	7 500
	No. 3/Stud	7.0		9.2		3.2	10 000	6 000
Spruce-Pine-Fir	SS	16.5		14.5		8.6	10 500	7 500
	No. 1/No. 2	11.8	1.5	11.5	5.3	5.5	9 500	6 500
	No. 3/Stud	7.0		9.0		3.2	9 000	5 500
Northern	SS	10.6		13.0		6.2	7 500	5 500
	No. 1/No. 2	7.6	1.3	10.4	3.5	4.0	7 000	5 000
	No. 3/Stud	4.5		5.2		2.0	6 500	4 000

Note: *Tabulated values are based on the following standard conditions:*
(a) 286 mm *larger dimension;*
(b) dry service conditions; and
(c) standard-term duration of load.

Beam Stability

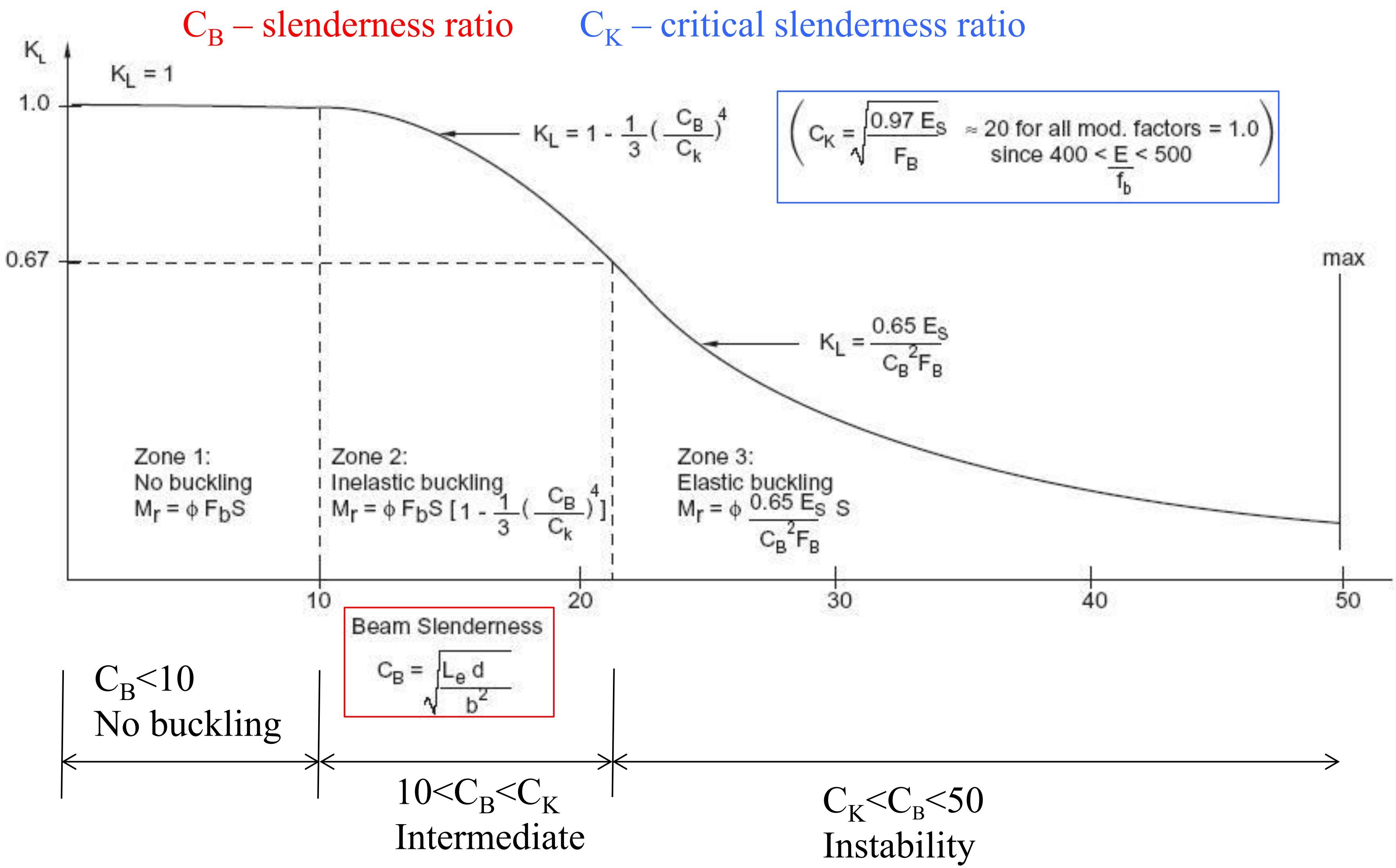
- Compression edge may buckle if not restrained – deep beam with long span
- Critical bending moment is



$$M_{cri} = \sqrt{\frac{\pi}{L_e}}\sqrt{\frac{EI_y}{GJ}}$$

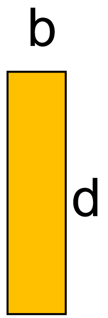
where
EI_y = bending stiffness in the lateral direction
GJ = St. Venant torsional rigidity
L_e = effective length

Basis of calculation procedure for K_L in Clause 7.5.6.2



Simplified rules for lumber - K_L=1

Based on depth to width (d:b) ratio of member



6.5.4.2 Lateral stability factor, K_L

6.5.4.2.1

The lateral stability factor, K_L, may be taken as unity when lateral support is provided at points of bearing to prevent lateral displacement and rotation, provided that the maximum depth-to-width ratio of the member does not exceed the following values:

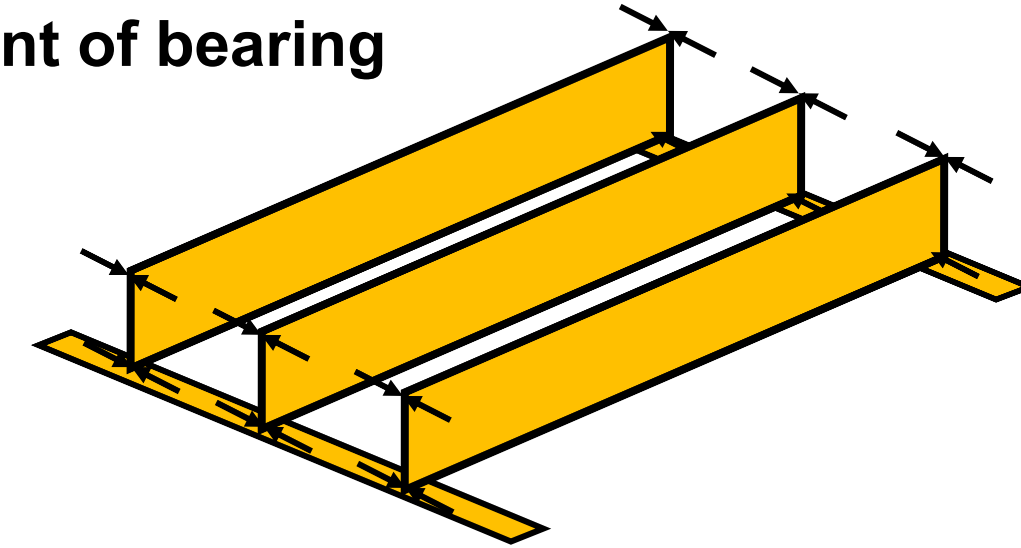
(a) 4:1 if no additional intermediate support is provided;
(b) 5:1 if the member is held in line by purlins or tie rods;
(c) 6.5:1 if the compressive edge is held in line by direct connection of decking or joists spaced not more than 610 mm apart;
(d) 7.5:1 if the compressive edge is held in line by direct connection of decking or joists spaced not more than 610 mm apart and adequate bridging or blocking is installed at intervals not exceeding eight times the depth of the member; or
(e) 9:1 if both edges are held in line.

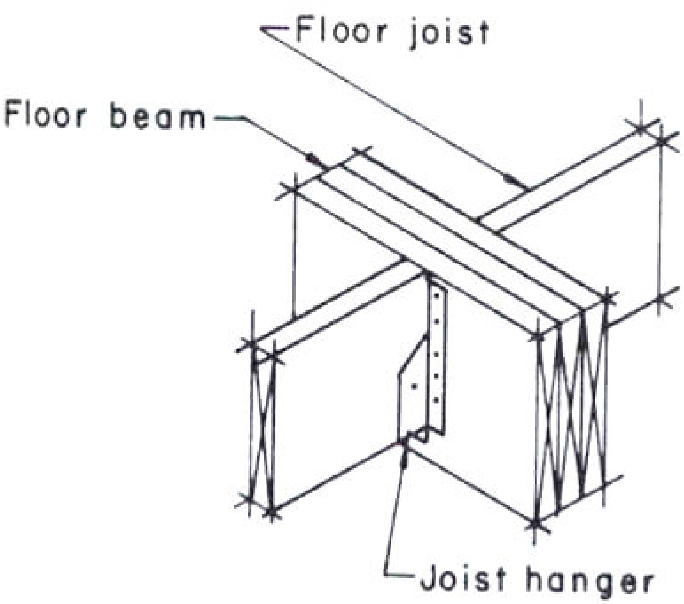
Alternatively, K_L may be calculated in accordance with Clause 7.5.6.4.

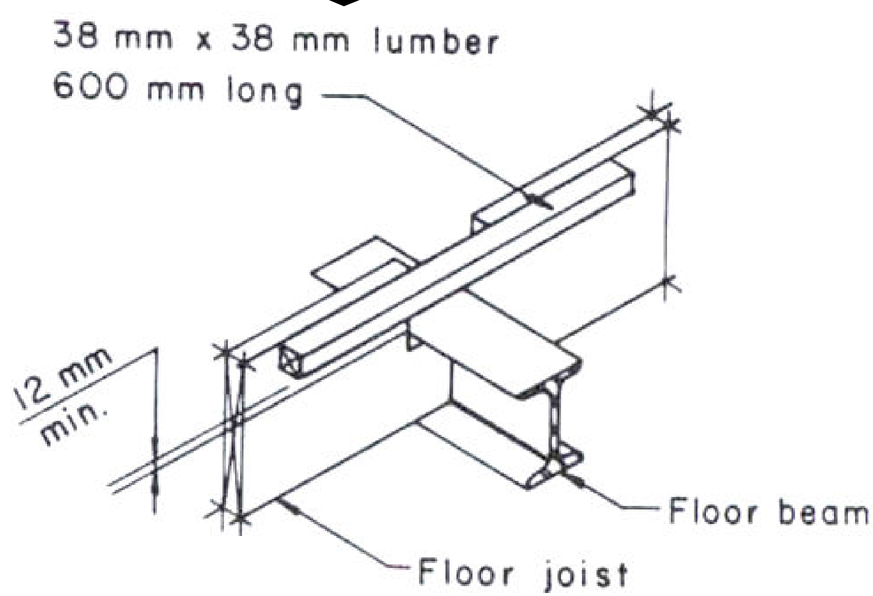
$K_L = 1$

For d:b up to 4:1

lateral supports at point of bearing





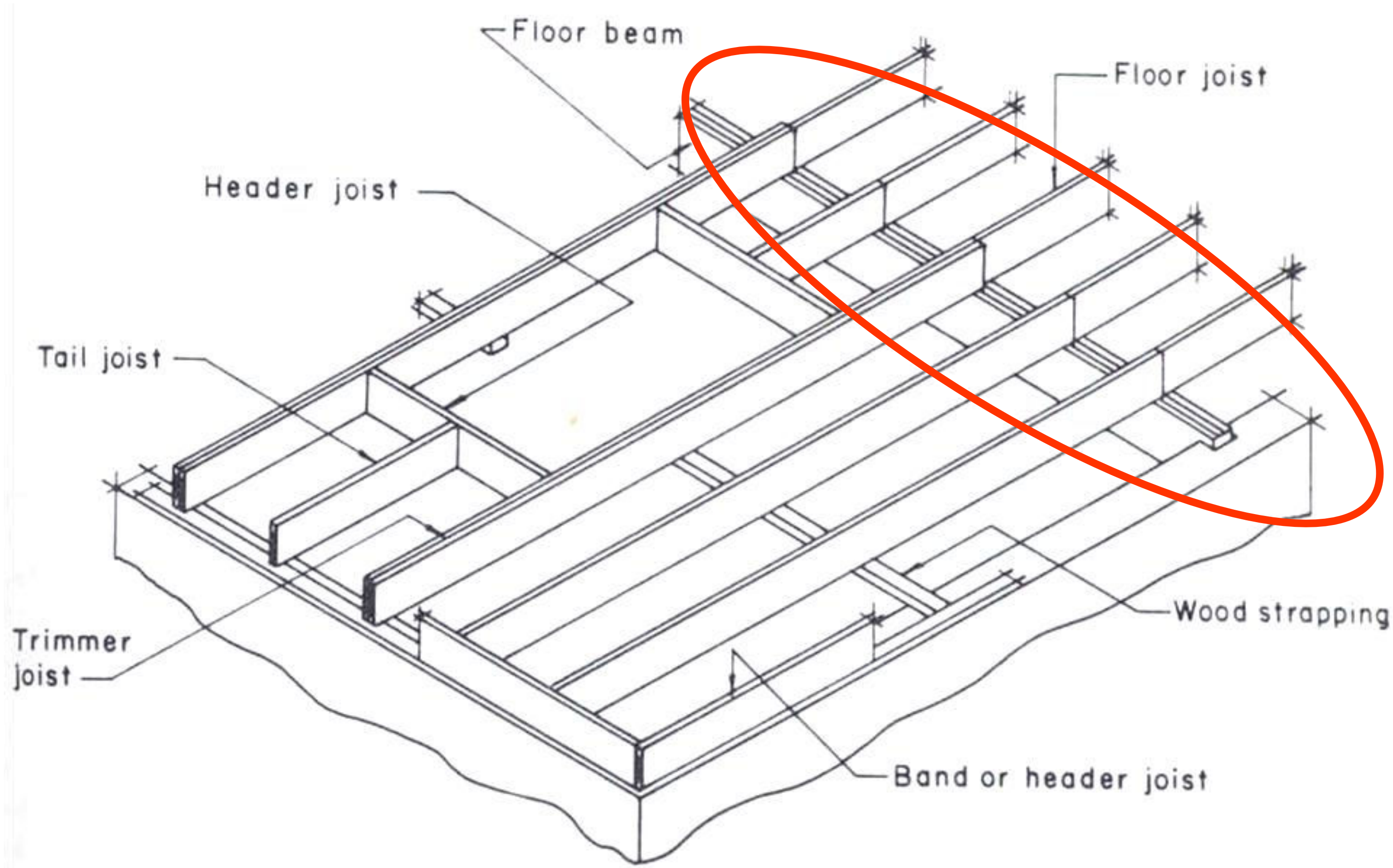


$K_L = 1$

For d:b up to 5:1

Lateral supports at point of bearing +

- the member is held in line by floor joists, purlins or tie-rods

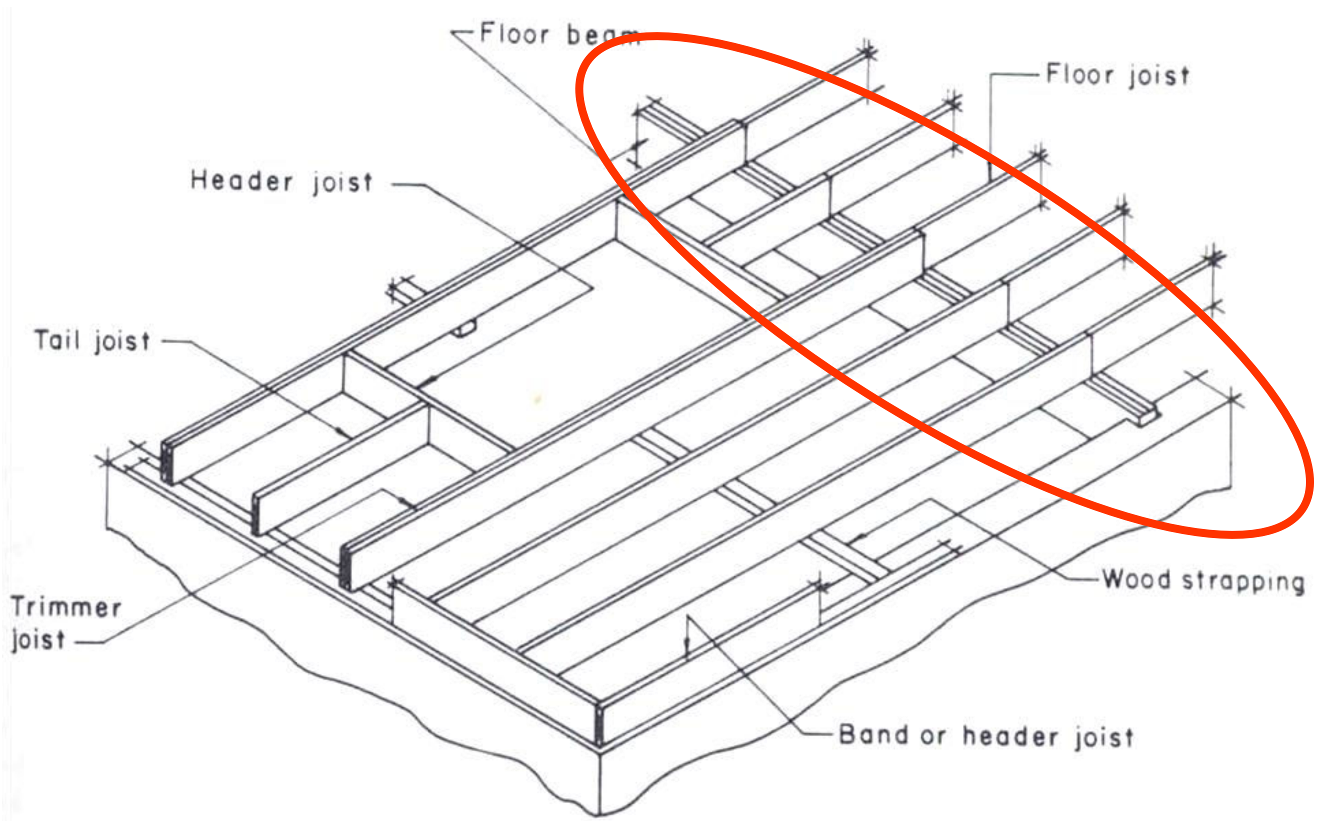


$K_L = 1$

For d:b up to 6.5:1

Lateral supports at point of bearing +

- the compressive edge is held in line by direct connection of decking or
- Joists/purlins spaced not more than 610 mm apart

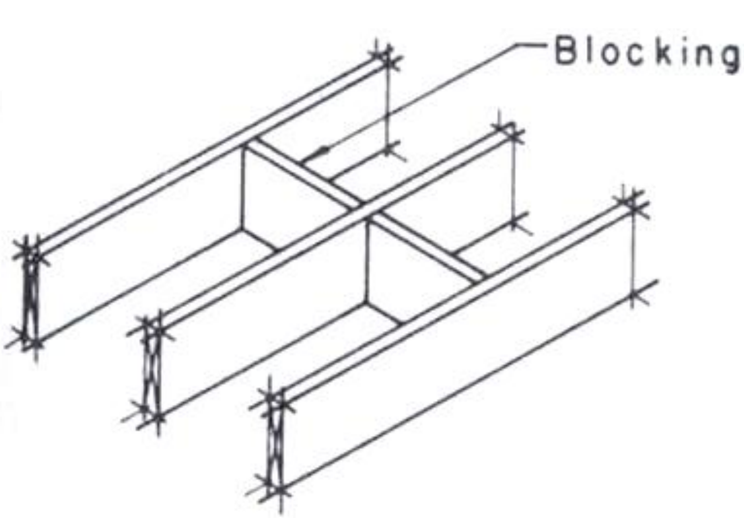


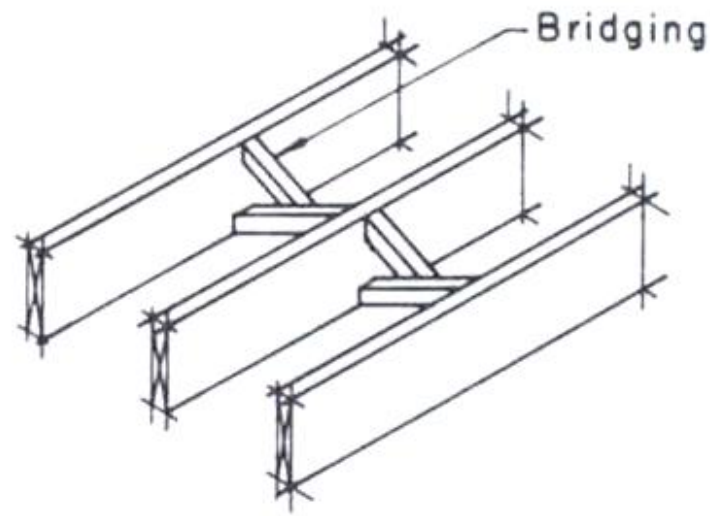
$K_L = 1$

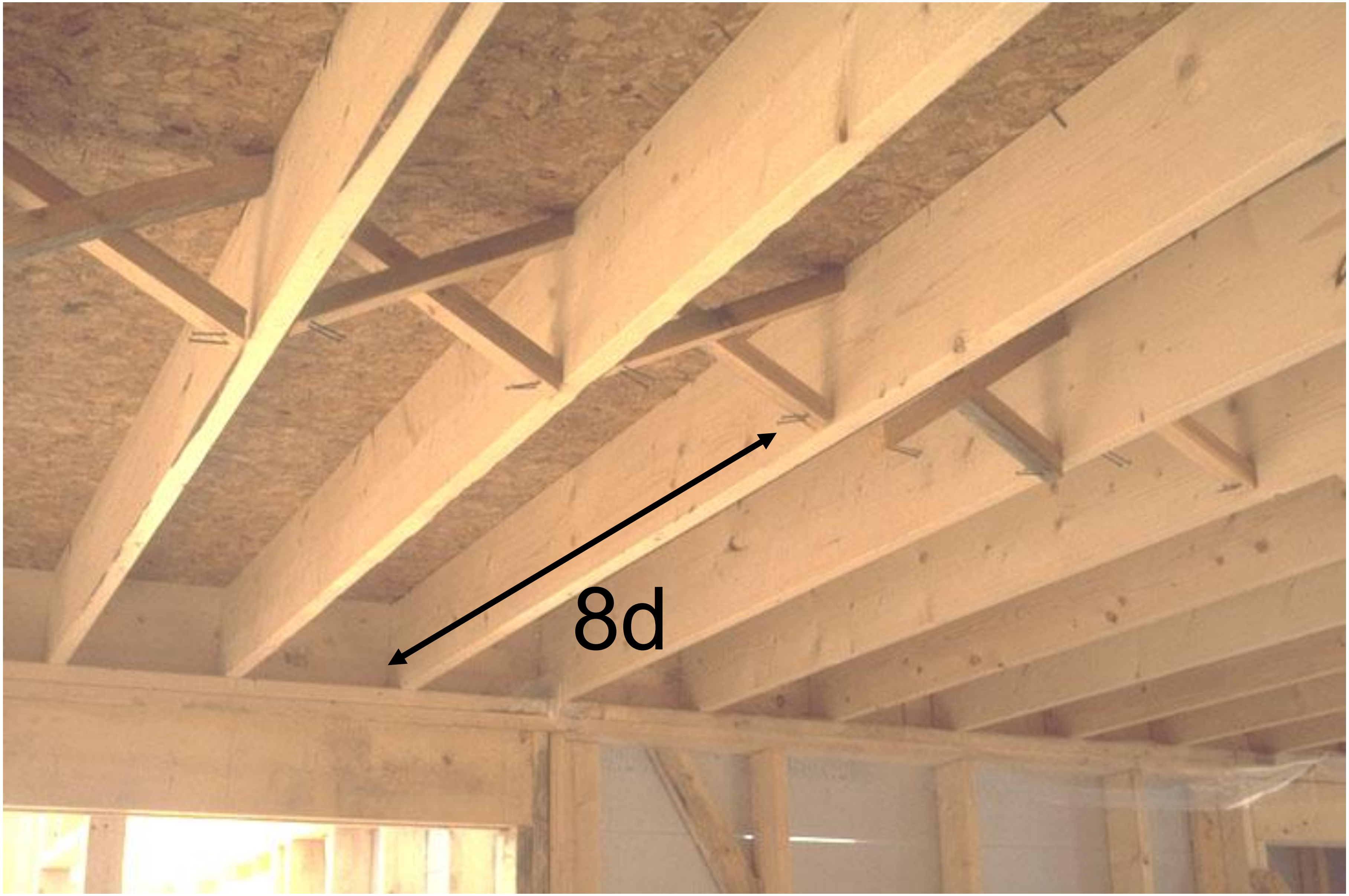
For d:b up to 7.5:1

lateral supports at point of bearing +

- the compressive edge is held in line by direct connection of decking or
- joists spaced not more than 610 mm apart and adequate bridging or blocking is installed at intervals not exceeding 8 times the depth of the member






8d

$K_L = 1$

For d:b up to 9:1

Both edges of the lumber member are held in place with adequate restraints provided to top and bottom edges e.g. subfloor and gypsum in floor construction

Design procedure in calculating moment resistance of lumber member

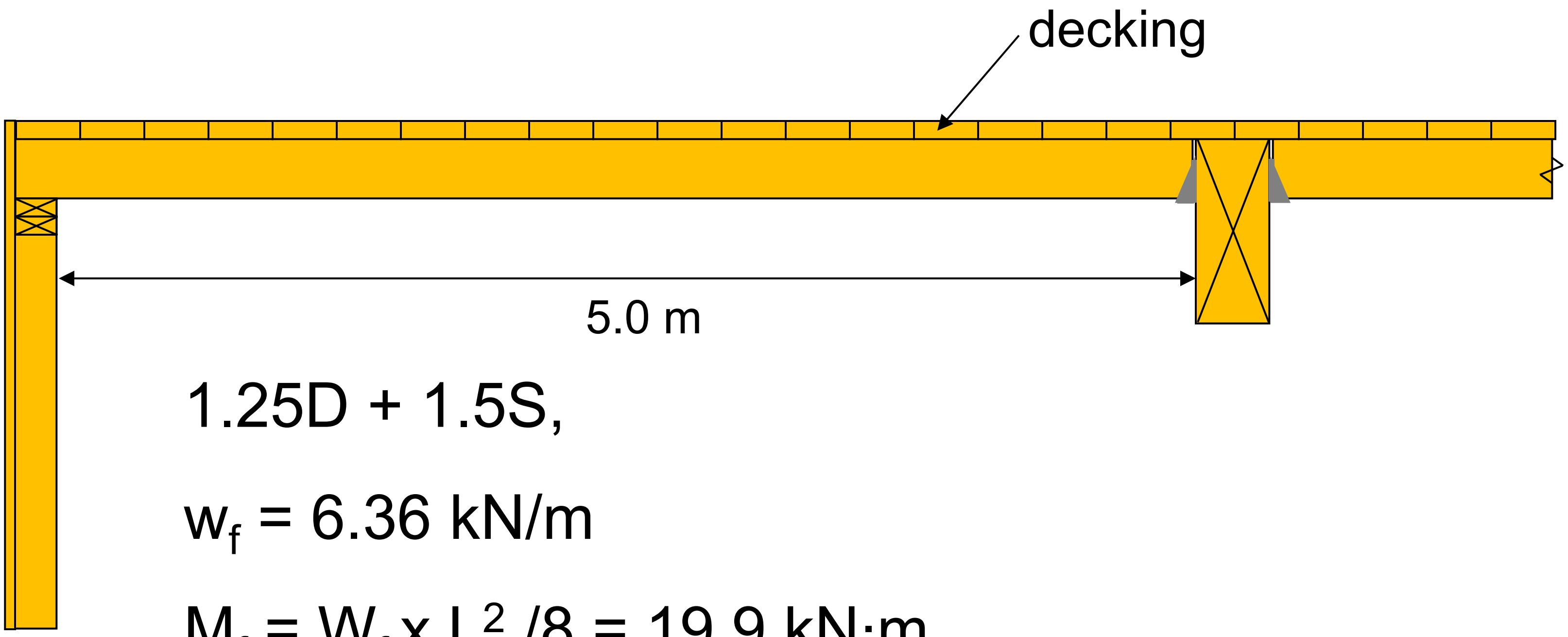
1. Determine the degree of lateral support and select a trial section based on K_L=1
2. Stocky beam - the design can be finalized
3. Slender or intermediate beam - calculate K_L and determine effective critical moment. Could be an iterative process

Ex 1: Lumber beam design

Douglas fir roof joist

D=0.75 kPa, S=2.2 kPa (ignore other loads)

trib. width = 1.5m



1.25D + 1.5S,

w_f = 6.36 kN/m

$M_f = W_f \times L^2 /8 = 19.9$ kN·m

Assume $K_D = K_H = K_{Sb} = K_T = 1.0$

- Acceptable d/b ratio = 6.5 since the compression edge is continuously supported
- Assume size to be used
  - Trial section : 140mm x 241mm (d/b=1.72 i.e. ok), K_{zb} = 1.2 for this size
- Determine grade category
  - Beam & Stringer
- Define grade
  - No. 1
- Obtain specified strength value
  - f_b = 15.8 MPa, Table 6.3.1C

Table 6.4.5
Size factor, K_Z, for visually stress-graded lumber

Larger dimension, mm	Bending and shear K_{Zb}, K_{Zv} — Smaller dimension, mm: 38 to 64	89 to 102	114 or more	Tension parallel to grain, K_{Zt}	Compression perpendicular to grain, K_{Zcp}	Compression parallel to grain, K_{Zc}	All other properties
				All	All	All	All
38	1.7	—	—	1.5	See Clause 6.5.7.5	Value computed using formula in Clause 6.5.6.2.3	1.0
64	1.7	—	—	1.5			1.0
89	1.7	1.7	—	1.5			1.0
114	1.5	1.6	1.3	1.4			1.0
140	1.4	1.5	1.3	1.3			1.0
184 to 191	1.2	1.3	1.3	1.2			1.0
235 to 241	1.1	1.2	1.2	1.1			1.0
286 to 292	1.0	1.1	1.1	1.0			1.0
337 to 343	0.9	1.0	1.0	0.9			1.0
387 or larger	0.8	0.9	0.9	0.8			1.0

Table 6.3.1C
Specified strengths and modulus of elasticity for beam and stringer grades, MPa

Species identification	Grade	Bending at extreme fibre, f_b*	Longi-tudinal shear, f_v	Compression: Parallel to grain, f_c	Compression: Perpen-dicular to grain, f_{cp}	Tension parallel to grain, f_t	Modulus of elasticity: E*	Modulus of elasticity: E_{05}*
D Fir-L	SS	19.5	1.5	13.2	7.0	10.0	12 000	8 000
	No. 1	15.8		11.0		7.0	12 000	8 000
	No. 2	9.0		7.2		3.3	9 500	6 000
Hem-Fir	SS	14.5	1.2	10.8	4.6	7.4	10 000	7 000
	No. 1	11.7		9.0		5.2	10 000	7 000
	No. 2	6.7		5.9		2.4	8 000	5 500
Spruce-Pine-Fir	SS	13.6	1.2	9.5	5.3	7.0	8 500	6 000
	No. 1	11.0		7.9		4.9	8 500	6 000
	No. 2	6.3		5.2		2.3	6 500	4 500
Northern	SS	12.8	1.0	7.2	3.5	6.5	8 000	5 500
	No. 1	10.8		6.0		4.6	8 000	5 500
	No. 2	5.9		3.9		2.2	6 000	4 000

$$M_r = \Phi\ F_b\ S\ K_{zb}\ K_L$$

$$M_r = 0.9 \cdot (15.8 \cdot 1 \cdot 1 \cdot 1 \cdot 1) \cdot \frac{140 \cdot 241^2}{6} \cdot 1.2 \cdot 1.0$$

= 23.13 kNm < 19.9 kNm = M_f

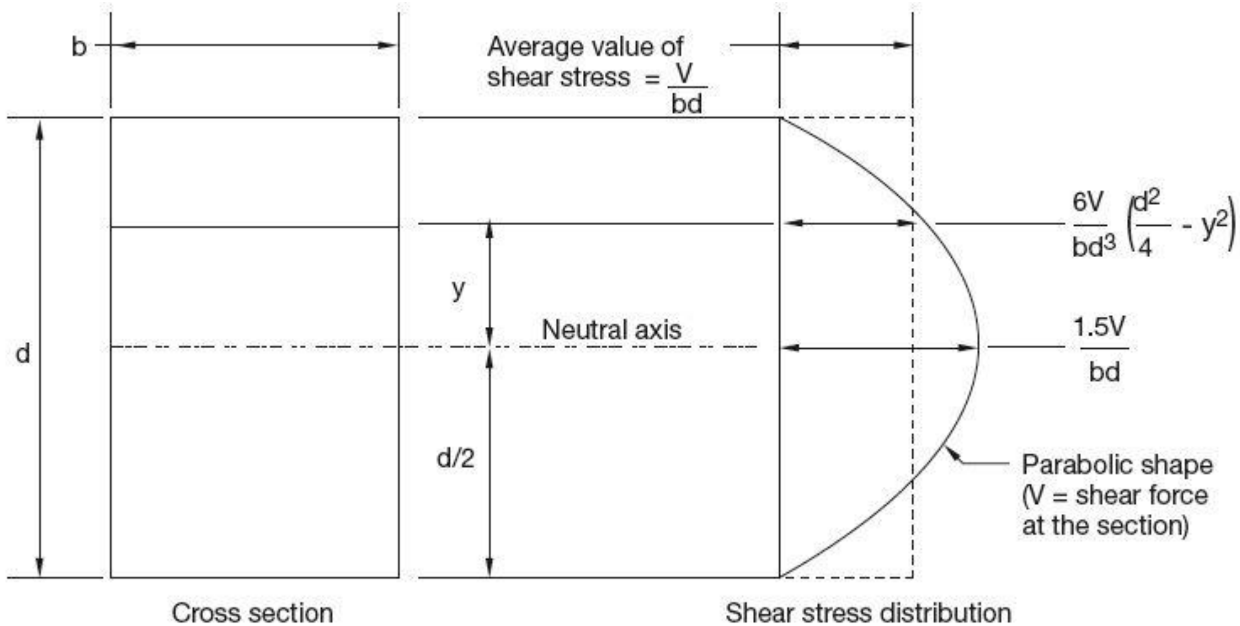
O.K.

Shear stress distribution in beam

In beam, shear stress at level y in a cross section subjected to vertical shear force, V:

$$\tau = \frac{VQ}{Ib}$$

where
Q = first moment of area
I = second moment of area
b = width of member at y



For rectangular cross section:

$$\tau_{\max} = \frac{3}{2}\tau_{Ave} = \frac{3V}{2A} \quad or \quad V = \tau_{\max}\frac{2}{3}A$$

Maximum shear stress occurs at neutral axis since Q is maximum

Shear resistance of sawn lumber member

6.5.5.2 Shear resistance

The factored shear resistance, V_r, shall not be less than the maximum factored shear force, V_f, and shall be taken as follows:

$$V_r = \phi F_v \frac{2A_n}{3} K_{Zv}$$

where

ϕ = 0.9

F_v = $f_v (K_D K_H K_{Sv} K_T)$

where

f_v = specified strength in shear, MPa (Clause 6.3)

A_n = net area of cross-section, mm^2 (Clause 5.3.8)

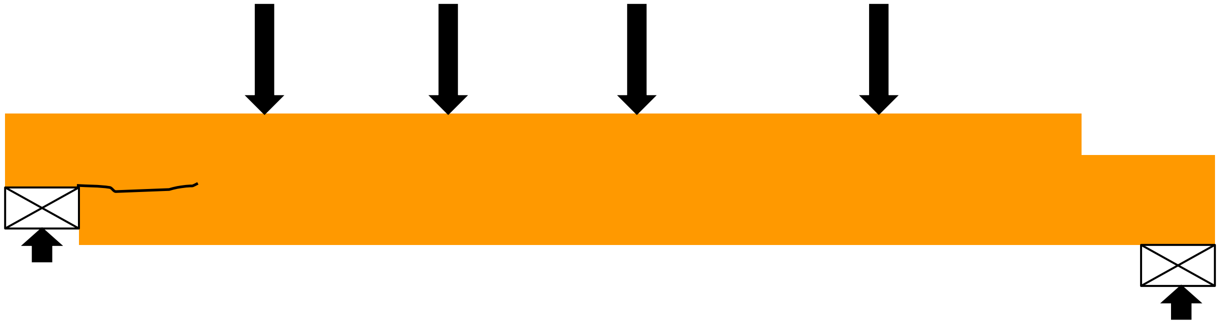
K_{Zv} = size factor in shear (Clause 6.4.5)

Net section, in case there is notch or an opening

Effect of all loads acting within a distance from a support equal to depth of member shall be ignored.

Beam with end notches

- To increase clearance
- To bring the top surface of the beam level with other beams
- Make room for pipes, framing of other beams
- Not permitted in high bending moment area



- Notches at tension edge can produce tension perpendicular to grain and encourage crack growth and are more critical notches at compression edge

Fracture shear force resistance at a notch on tension side at support - 6.5.5.3

For a member with a notch on the tension face, an additional capacity check must be made concerning avoidance of fracture at a re-entrant corner of a notch, known as *factored fracture shear resistance*,

$$F_r = \phi F_f A_g K_N$$

→ *Based on fracture mechanics principles*

where $\phi = 0.9$

$F_f = f_f\,(K_D K_H K_{Sf} K_T)$, MPa

$f_f = 0.5$ MPa for all sawn members

A_g = Gross cross-section, mm^2

K_N = Notch Factor § 6.5.5.3.2

Notch Factor 6.5.5.3.2

For members with a rectangular section

$$K_N = \left[0.006d\left(1.6\left(\frac{1}{\alpha}-1\right)+\eta^2\left(\frac{1}{\alpha^3}-1\right)\right)\right]^{-\frac{1}{2}}$$

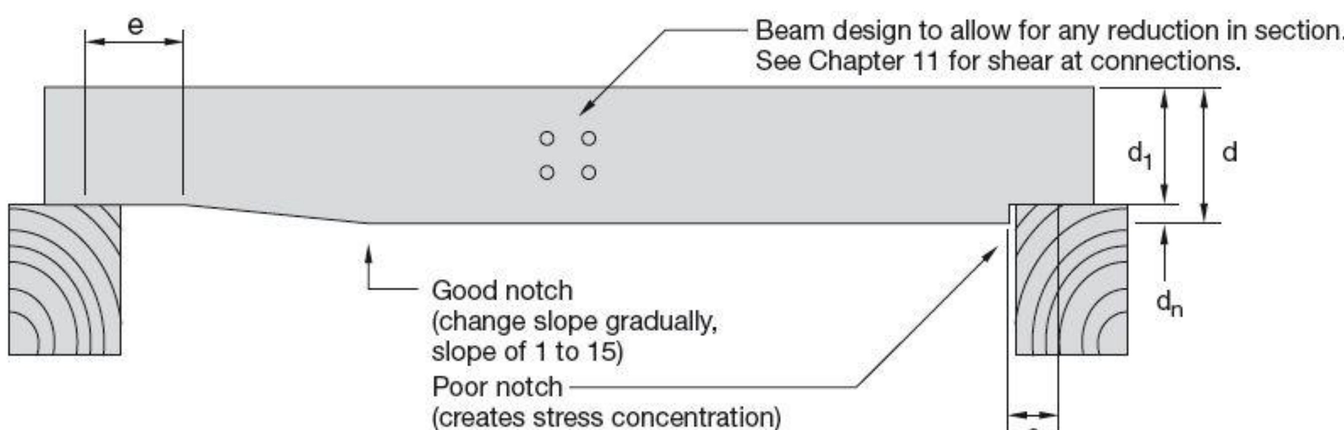
where d = depth of cross-section, mm

$\alpha = 1 - (d_n/d)$

d_n = depth of notch measured normal to member axis ≤0.25d

$\eta = e/d$

e = length of notch measured parallel to the member axis, from the centre of the nearest support to the re-entrant corner of notch, mm.



Determination of length and depth of notch

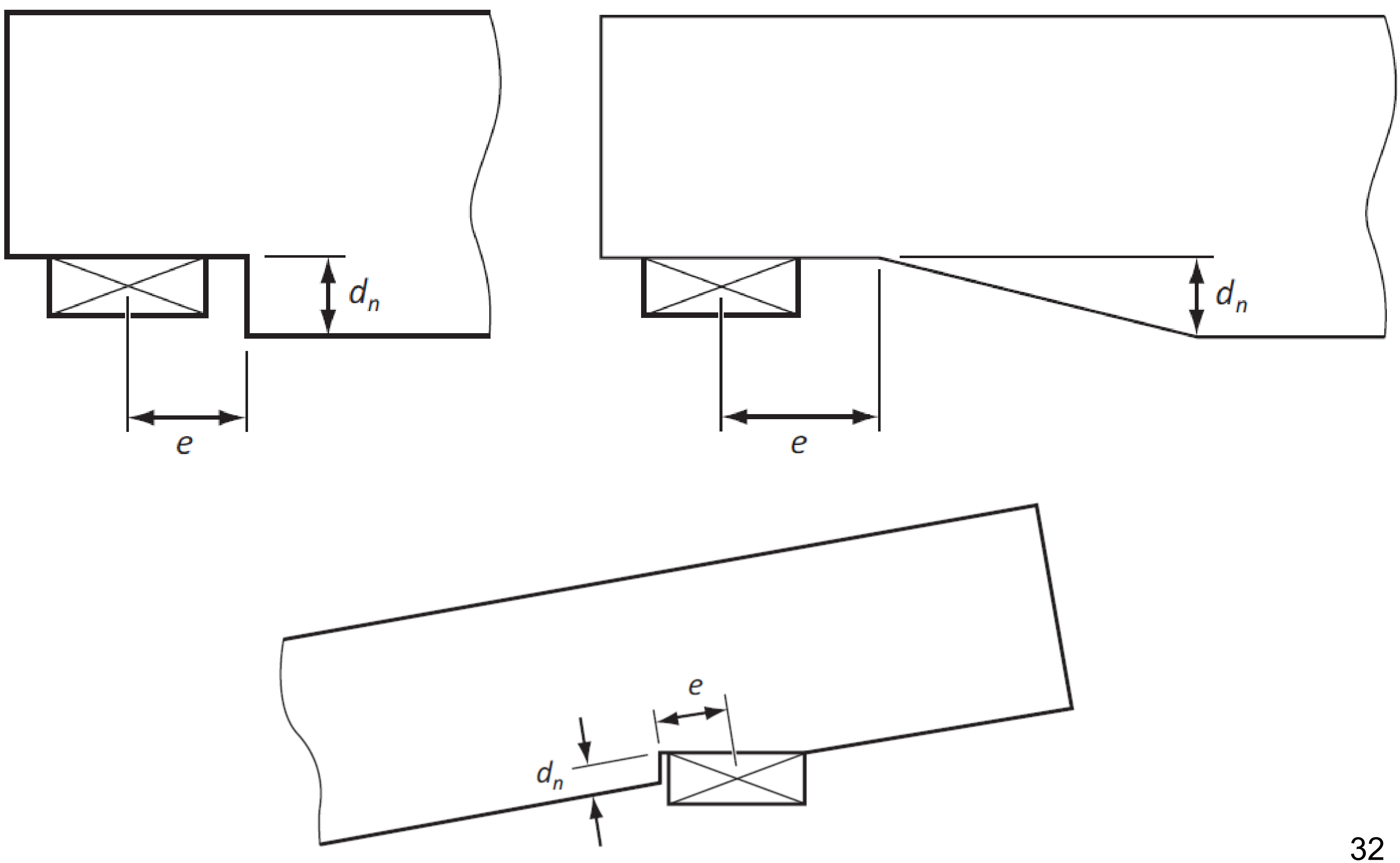


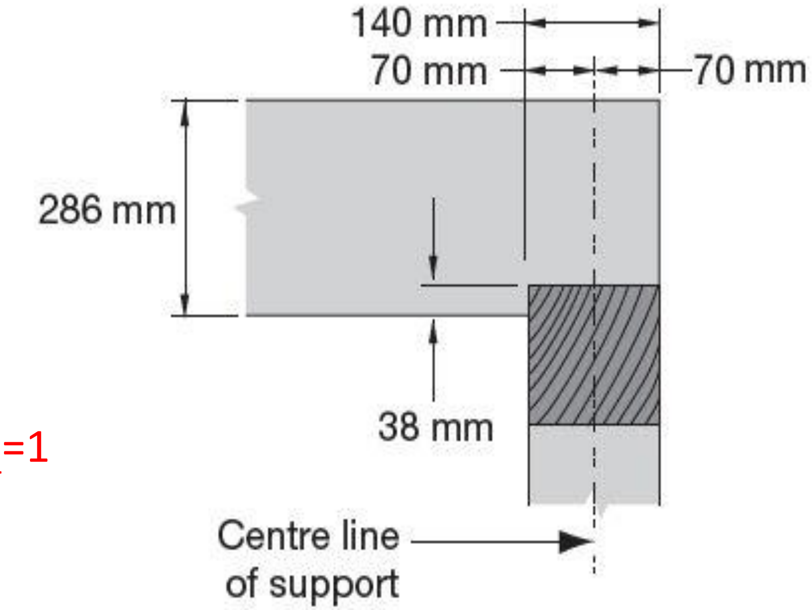
Table 6.5.5.3.2
Values of $K_N\sqrt{d}$

	α				
η	0.75	0.80	0.85	0.90	0.95
0.15	17.2	19.9	23.7	29.9	43.5
0.20	16.8	19.5	23.3	29.4	42.8
0.25	16.4	19.0	22.8	28.8	42.0
0.30	15.9	18.5	22.2	28.1	41.0
0.35	15.4	18.0	21.5	27.3	39.9
0.40	14.9	17.4	20.9	26.5	38.8
0.45	14.3	16.8	20.2	25.7	37.6
0.50	13.8	16.2	19.5	24.8	36.4
0.60	12.7	15.0	18.1	23.1	34.0
0.70	11.8	13.9	16.8	21.5	31.7
0.80	10.9	12.8	15.6	20.0	29.6
0.90	10.1	11.9	14.5	18.7	27.6
1.00	9.36	11.1	13.5	17.4	25.8
1.20	8.15	9.70	11.8	15.3	22.7
1.40	7.20	8.57	10.5	13.6	20.2
1.60	6.42	7.66	9.39	12.1	18.1
1.80	5.79	6.91	8.48	11.0	16.4
2.00	5.26	6.29	7.72	10.0	14.9

Ex. 2 - Joist notched on tension side at support

Verify that the Hem-Fir No.1/No.2 38 × 286 mm floor joists (see drawing) notched at the supports are adequate for the following conditions:

- joist spacing = 600 mm
- joist span = 4.0 m
- specified dead load = 1.2 kPa
- specified live load = 2.4 kPa
- standard load duration K_D=1
- dry service conditions K_S=1, K_{Sf}=1
- untreated K_T=1
- fully laterally supported by subfloor K_L=1
- Case 2 system K_H=1.4



Ex. 2 - Solution

- Calculation:
  - Total factored load (ultimate limit states)

    = 1.25D+1.5L = (1.25 x 1.2)+(1.5 x 2.4) = 5.10 kPa
  - Total specified load (serviceability limit states)

    = D + L = 1.20 + 2.40 = 3.60 kPa

  - Design load on one joist:

    w_f = 5.10 x 0.6 = 3.06 kN/m

    w = 3.60 x 0.6 = 2.16 kN/m

    w_L = 2.40 x 0.6 = 1.44 kN/m

Ex. 2 - Solution

- Factored moment:
  - $M_f = W_f L^2 / 8 = (3.06 \times 4^2)/8 =$ **6.12 kNm**
- Factored shear:
  - $V_f = W_f L / 2 = (3.06 \times 4)/2 =$ **6.12 kN**

  (same as factored bearing)
- Required bending stiffness:
  - $\Delta = (5 W_L \times L^4) / (384 E_s I) \leq L/360$
  - Required $E_s I =$ **432×10^9 Nmm²**

Ex. 2 - Solution

- Check bending moment:

  M_r = 7.17 kNm > 6.12 kNm $\therefore OK$

- Check shear in cross section over notch:

  A_N = (286-38) x 38 = 9424 mm^2

  F_v = 1.6 x (1 x 1.4 x1 x 1) = 2.24 MPa

  V_r = 12.7 kN > 6.12 kN $\therefore OK$

- Check deflection:

  E_sI = 815 x 10^9 > 432 x 10^9 Nmm^2 $\therefore OK$

Ex. 2 - Solution

- Factored fractured resistance at notch,

$$F_r = \phi\ F_f\ A_g K_N$$

$F_f = 0.5 \times 1.4 = 0.7$ MPa

$A_g = 10868\ mm^2$

$\alpha = 1 – 38/286 = 0.86$

$e = 70$ mm

$\eta = 70 / 286 = 0.24$

$K_N \sqrt{d} = 24.5$ (Table 6.5.5.3.2), $K_N = 1.45$

$$F_r = 0.9 \times 0.7 \times 10868 \times 1.45 = 9.93 \text{ kN}$$

$$> V_f = 6.12 \quad \therefore OK$$

Bearing resistance at beam support

Effect of all loads acting on the beam (6.5.7.2):

$$Q_r = \phi F_{cp} A_b K_B K_{Zcp}$$

where Φ =0.8

$F_{cp}=f_{cp}(K_D K_{Scp} K_T)$, MPa

A_b = Bearing area, mm^2

K_B = Length of bearing factor (§ 6.5.7.6)

K_{Zcp} = Size factor for bearing (§ 6.5.7.4)

Bearing size factor

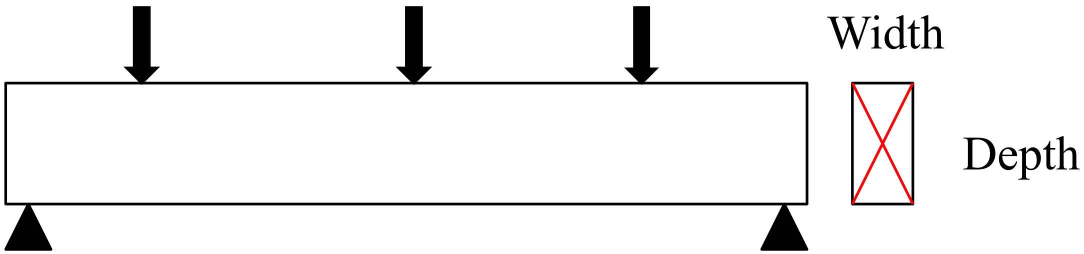


Table 6.5.7.4
Size factor for bearing, K_{Zcp}

Ratio of member width to member depth*	K_{Zcp}
1.0 or less	1.00
2.0 or more	1.15

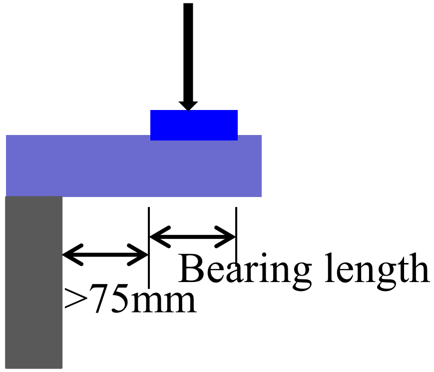
**Interpolation applies for intermediate ratios.*

Increase capacity when width is greater than depth ie loaded on flat

Length of bearing factor

Table 6.5.7.5
Length of bearing factor, K_B

Bearing length (parallel to grain) or washer diameter, mm	Modification factor, K_B
12.5 and less	1.75
25.0	1.38
38.0	1.25
50.0	1.19
75.0	1.13
100.0	1.10
150.0 or more	1.00



Capacity is increased when bearing length is less than 150mm.
Conditions for increase are:

(a) no part of the bearing area is less than 75 mm from the end of the members; and
(b) bearing areas do not occur in positions of high bending stresses.

Bearing resistance at beam support

Effect of loads applied near a support only (6.5.7.3)

$$Q'_r = \tfrac{2}{3}\phi F_{cp} A'_b K_B K_{Zcp}$$

where Φ = 0.8

$F_{cp} = f_{cp}(K_D K_{Scp} K_T)$, MPa

A'_b = Average bearing area, mm^2

$$A'_b = b\left(\frac{L_{b1} + L_{b2}}{2}\right) \quad \leq 1.5b(L_{b1})$$

b = average bearing width perp. grain, mm

L_{b1} = lesser bearing length, mm

L_{b2} = larger bearing length, mm

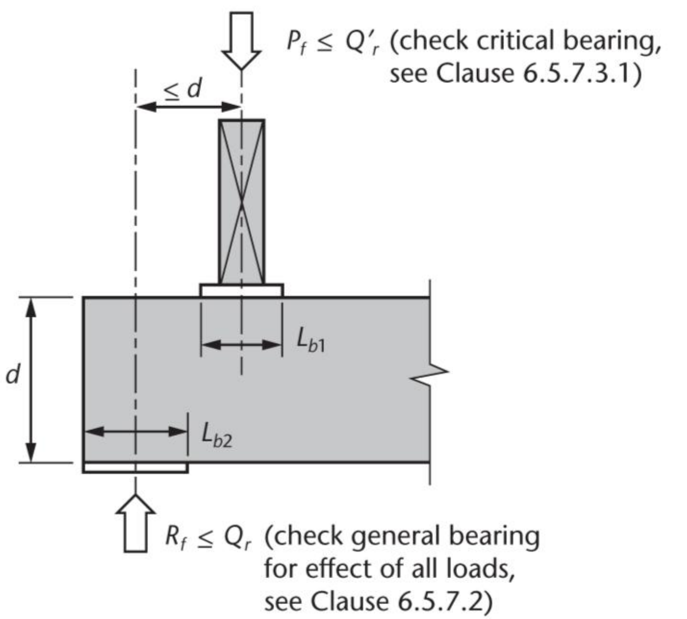


Figure 6.5.7.3
Load applied near a support

Ex. 2 (slide 34) – Solution

- Check factored bearing, $Q_r = \phi F_{cp} A_b K_B K_{Zcp}$

$F_{cp} = f_{cp}(K_D K_{Scp} K_T) = 4.6 \times (1 \times 1 \times 1) = 4.6$ MPa
$A_b = 38 \times 140 = 5320\ mm^2$
$K_B = 1$ (§ 6.5.7.6)
$K_{Zcp} = 1$ (§ 6.5.7.4)

$Q_r = 0.8 \times 4.6 \times 5320 \times 1 \times 1 = 19.6$ kN
$> Q_f = 6.12$ kN $\quad \therefore OK$

Serviceability Limit States

- Deflections shall be limited
  - to ensure buildings are serviceable and finishing materials are not damaged
  - to avoid poor fit over doors, windows and partitions
- Ponding (Clause 5.4.4)
- Vibrations (Clause 5.4.5)

Deflection Criteria

Δ = Deflection due to specified load (UDL)

$$\Delta = \frac{5}{384}\frac{WL^4}{E_s I}$$

$E_s = E(K_{SE}K_T)$

E = Modulus of elasticity, MPa

K_{SE} = Service condition factor

K_T = Treatment factor

Δ_a = Allowable Deflection (§ 5.4.2 and 5.4.3)

= L/180 (total specified loads)

= L/360 (long-term load, when it is > 50% of total load)

Suggested deflection limits for different types of member

CWC Wood Design Manual

Table 2.1
Deflection Criteria

		Loading	Δ_{max}	Limitation
Roofs and floors		Total load	L/180	CSA O86
Plastered or gypsum ceilings:	Glulam	Live load	L/360	Suggested
	Lumber	Total load	L/360[1]	Suggested
Roofs		Snow load	L/240[2]	Suggested
Floors		Live load[3]	L/360	Suggested
Wind columns		Wind load	L/180	Suggested

Notes:
1. Part 9 of the *NBCC* permits L/360 deflection limitation based on live load for all roofs and floors with plaster or gypsum board.
2. In Part 9, this is required for roofs with ceilings other than plaster or gypsum. Where no ceilings exist, L/180 based on live load is permitted.
3. For floor beams supporting floors with concrete topping, L/360 based on total specified load is recommended.
4. For curved glulam members, refer to Section 9.2 and clause 4.5.2 of CSA O86.

Shear component in beam deflection

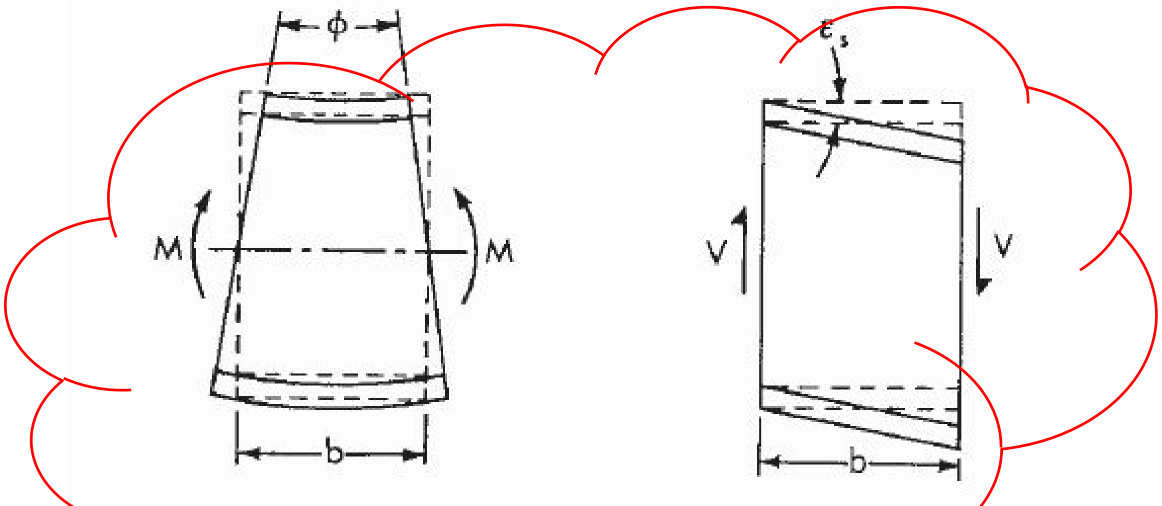


FIG. 1 Deflection in beam caused by bending moment, left, and by shear, right.

$$\Delta_{total} = \Delta_b + \Delta_s$$

- Wood has a low shear modulus compared with modulus of elasticity (E/G ≈ 16 for solid lumber)
- Total beam deflection (Δ_{total}) should include both bending (Δ_b) and shear (Δ_s)

Beam deflection equations with shear

Simply supported beam under UDL

$$\Delta_{total} = \frac{5}{384}\frac{WL^4}{E_s I} + \frac{WL^2}{8\mu GA}$$

Simply supported beam with point load at mid-span

$$\Delta_{total} = \frac{1}{48}\frac{PL^3}{E_s I} + \frac{PL^2}{4\mu GA}$$

where μ = shear coefficient of cross section shape (for rectangular cross section = 5/6)

Note:

- Lumber and glulam beams are not required to consider shear deflection because the modulus of elasticity that was measured to develop their properties already account for shear deflection (ie apparent modulus and not shear-free modulus)
- For EWP shear deflection shall be considered

Glued-laminated timber (Glulam) member

Species groups and grades

Table 7.2.1
Glued-laminated timber stress grades

	Wood species		
Primary application	**Douglas Fir-Larch**	**Spruce-Lodgepole Pine-Jack Pine**	**Hem-Fir and Douglas Fir-Larch**
Bending members	20f-E, 24f-E 20f-EX, 24f-EX	20f-E 20f-EX	24f-E 24f-EX
Compression members	16c-E	12c-E	
Tension members	18t-E	14t-E	

2000psi

flexure

tension

compression

Glulam bending members

- The stiffest and strongest laminations are located in the outer portions
- Grades:
  - 20f-E and 24f-E
    - Laminations on the tension face are stronger than those on the compression face
    - Used when no reverse bending moments are expected
  - 20f-EX and 24f-EX
    - Identical high grade laminations on both the compression and tension faces
    - Used when there are reverse bending moments

Design of glulam beam

1.	Factored bending moment resistance $M_r \geq M_f$	§ 7.5.6.5
2.	Factored shear resistance $V_r \geq V_f$	§ 7.5.7
3.	Maximum deflection ≤ Deflection criteria	§ 5.5
4.	Factored bearing resistance $Q_r \geq Q_f$	§ 7.5.9
5.	Notches	§ 7.5.7.4

An additional aspect of Glulam bending is the influence of curvature (not covered in this course)

Moment resistance

7.5.6.5 Moment resistance

7.5.6.5.1

Except as provided for in Clauses 7.5.6.5.3 and 7.5.6.6, the factored bending moment resistance, M_r, of glued-laminated timber members shall be taken as the lesser of M_{r1} or M_{r2}, as follows:

$$M_{r1} = \phi F_b S K_x K_{Zbg}$$

$$M_{r2} = \phi F_b S K_x K_L$$

The resistance may be governed by material strength or lateral stability.

where S = Section modulus, mm^3

ϕ = 0.9

$F_b = f_b(K_D K_H K_{Sb} K_T)$

where

f_b = specified strength in bending, MPa (Table 7.3)

K_X = curvature factor (Clause 7.5.6.5.2)

K_X = 1 for straight member

$$K_{Zbg} = \left(\frac{130}{b}\right)^{\frac{1}{10}} \left(\frac{610}{d}\right)^{\frac{1}{10}} \left(\frac{9100}{L}\right)^{\frac{1}{10}} \leq 1.3$$

where

b = beam width (for single-piece laminations) or the width of widest piece (for multiple-piece laminations), mm

d = beam depth, mm

L = length of beam segment from point of zero moment to point of zero moment, mm

K_L = lateral stability factor (Clause 7.5.6.4)

Size (volume) factor, K_{Zbg}

$$K_{Zbg} = \left(\frac{130}{b}\right)^{\frac{1}{10}} \left(\frac{610}{d}\right)^{\frac{1}{10}} \left(\frac{9100}{L}\right)^{\frac{1}{10}} \leq 1.3$$

Volume effect factor
Reference beam 130mm x 610mm with span of 9.1m
< 1 if volume is greater, > 1 if smaller

where

b = beam width (for single-piece laminations) or the width of widest piece (for multiple-piece laminations), mm

d = beam depth, mm

L = length of beam segment from point of zero moment to point of zero moment, mm

- For beams with one or more points of inflections (i.e., multiple-span beams or cantilevered beams), the size factor is calculated for each beam segment. The moment resistance for each beam segment, as modified by the appropriate size factor, is then compared to the maximum factored moment within that segment. (note of § 7.5.6.5.1)

Curvature factor, K_X

7.5.6.5.2

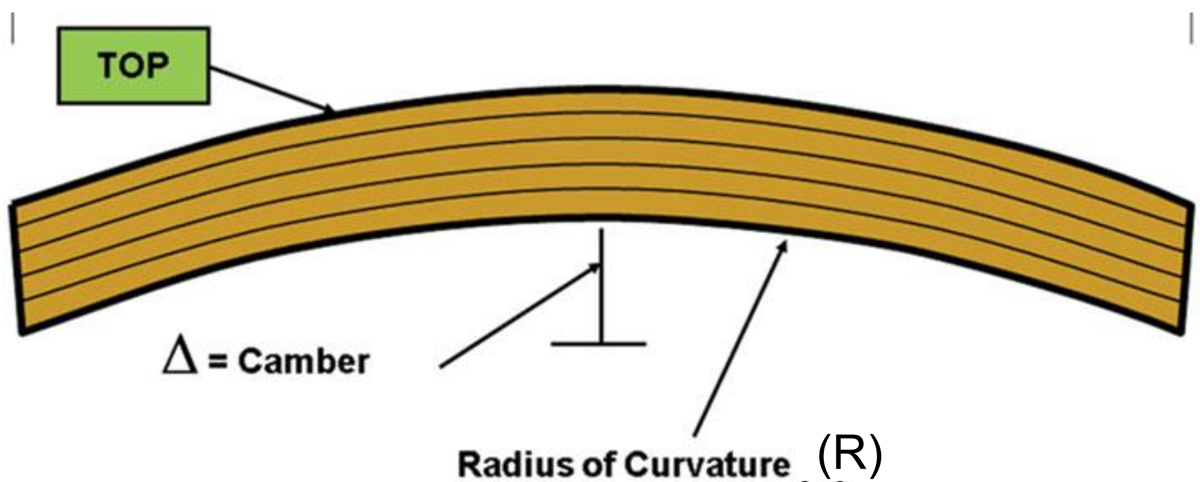
For the curved portion only of glued-laminated timber members, the specified strength in bending shall be multiplied by the curvature factor, taken as follows:

$$K_X = 1 - 2000\left(\frac{t}{R}\right)^2$$

where

t = lamination thickness, mm

R = radius of curvature of the innermost lamination, mm



Curved Beam, Pure Bending

- The neutral axis shift a distance *e* from the centroid axis.

$$e = r_c - \frac{A}{\int \frac{dA}{r}}$$

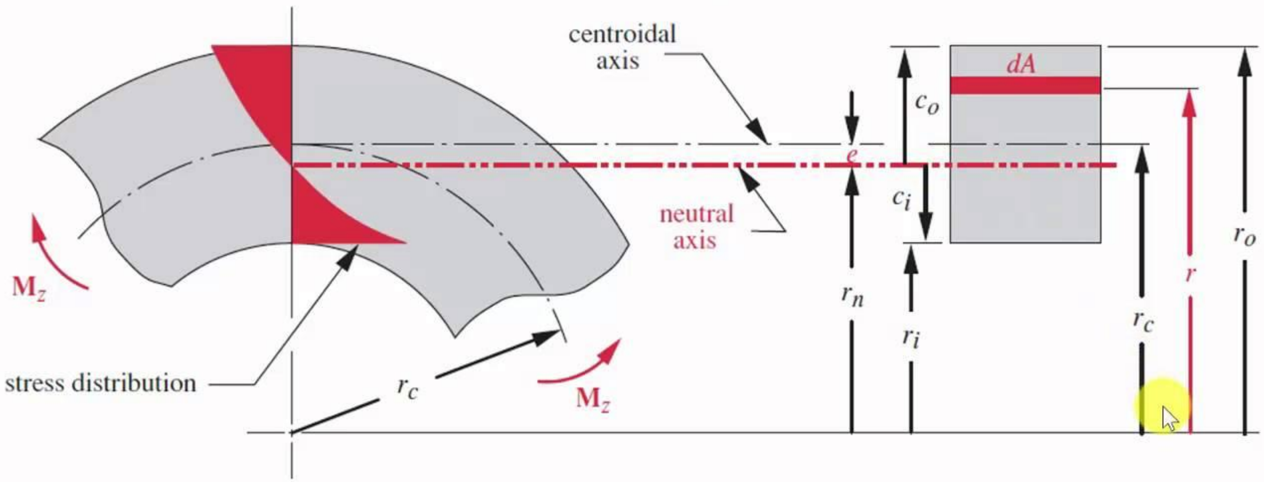
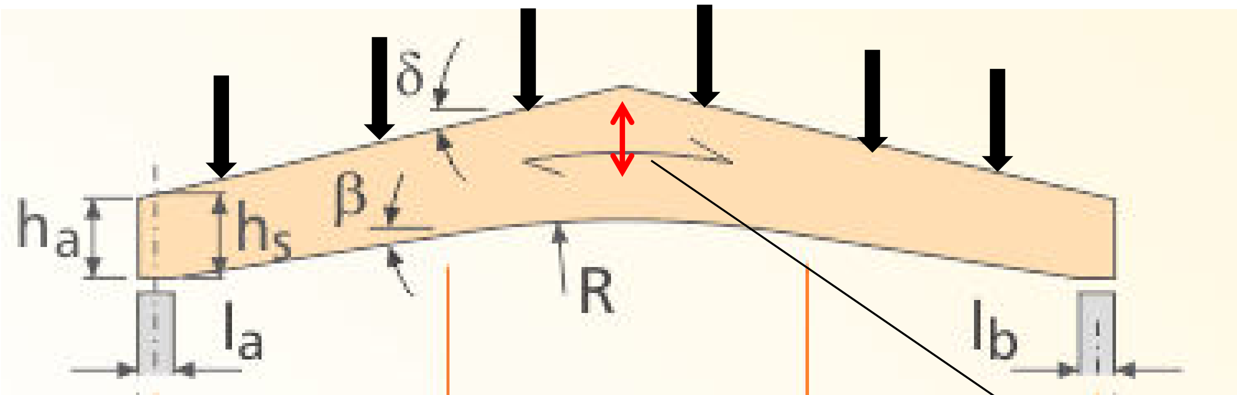


FIGURE 4-16

Segment of a Curved Beam in Pure Bending

Radial stresses in curved glulam beam



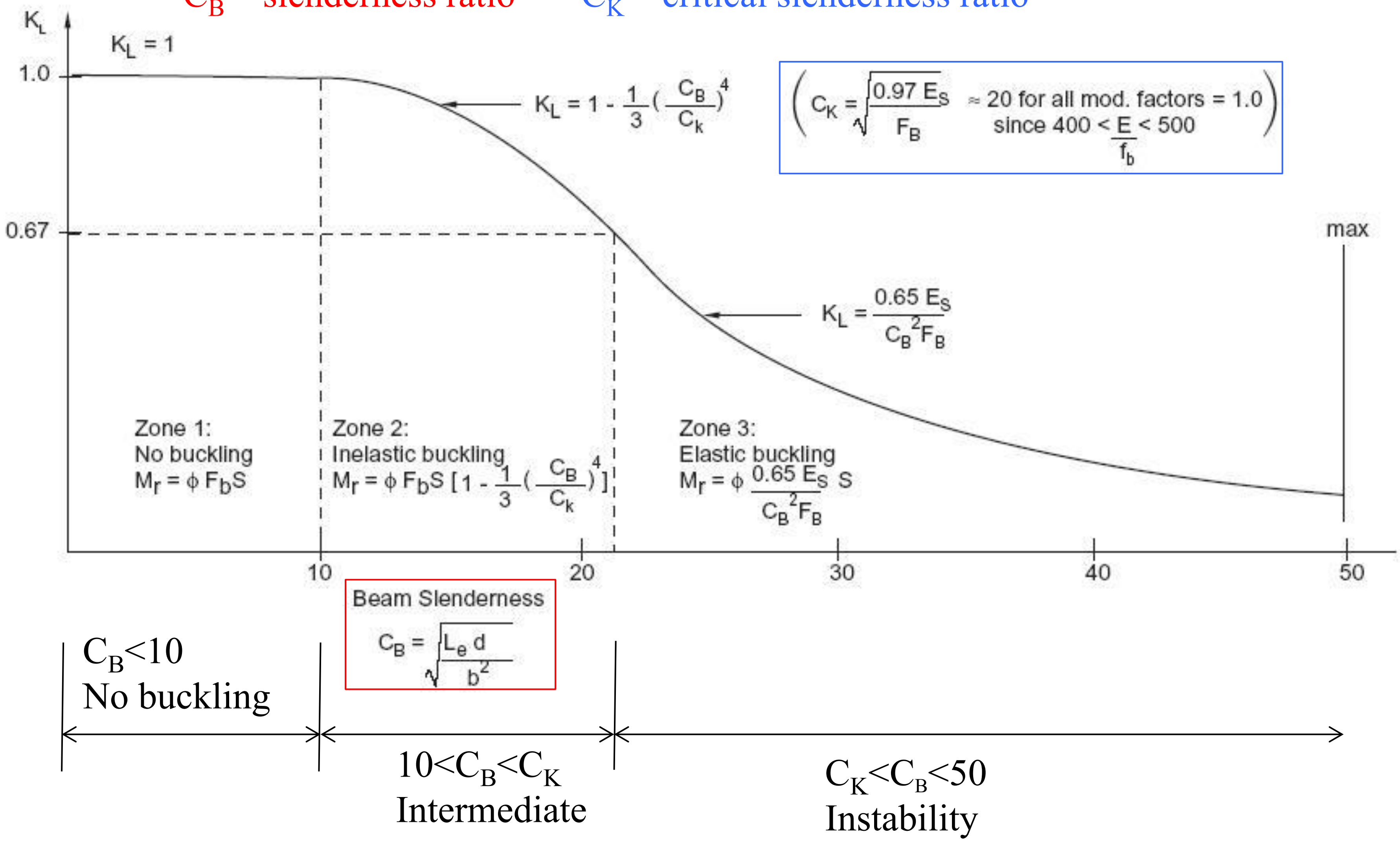
Radial stresses can cause beam to fail in tension perpendicular to grain and may govern moment capacity (Clause 7.5.6.6)

Lateral Stability Factor - K_L

- If d/b ratio ≤ 2.5:1, K_L= 1
- If d/b ratio > 2.5:1, use 7.5.6.4 to calculate K_L, value depending on degree of lateral restraint provided
- If compression edge is continuously supported (e.g. by decking) K_L=1 (see 7.4.6.4.2)

Basis of calculation procedure for K_L in Clause 7.5.6.4

C_B – slenderness ratio C_K – critical slenderness ratio

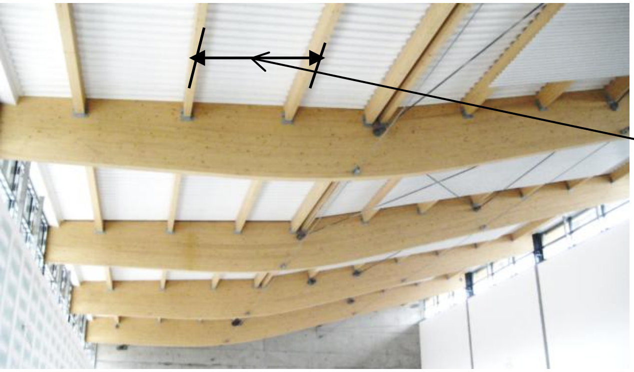


Effective length, L_e

- Unsupported length l_u (§ 7.5.6.4.1)
  - When no additional intermediate support is provided
    - l_u = the distance between points of bearing or the length of the cantilever
  - When intermediate support is provided by purlins and they prevent lateral displacement of the compressive edge
    - l_u = maximum purlin spacing, *a*
- Slenderness Ratio (§ 7.5.6.4.3)

$$C_B = \sqrt{\frac{L_e d}{b^2}} \leq 50$$

where d is beam depth and b is beam width



a = purlin spacing

l_u = distance between points of bearing support

Table 7.5.6.4.3
Effective length, L_e, for bending members

	Intermediate support	
	Yes	No
Beams		
Any loading	1.92a	1.92ℓ_u
Uniformly distributed load	1.92a	1.92ℓ_u
Concentrated load at centre	1.11a	1.61ℓ_u
Concentrated loads at 1/3 points	1.68a	
Concentrated loads at 1/4 points	1.54a	
Concentrated loads at 1/5 points	1.68a	
Concentrated loads at 1/6 points	1.73a	
Concentrated loads at 1/7 points	1.78a	
Concentrated loads at 1/8 points	1.84a	
Cantilevers		
Any loading		1.92ℓ_u
Uniformly distributed load		1.23ℓ_u
Concentrated load at free end		1.69ℓ_u

Note: *ℓ_u and a are as defined in Clause 7.5.6.4.1.*

Slenderness ratio, C_B and critical slenderness ratio, C_k

a) If $C_B \leq 10$, then $K_L = 1.0$

$$C_B = \sqrt{\frac{L_e d}{b^2}} \leq 50$$

b) If $10 < C_B \leq C_K$, then

$$K_L = 1 - \frac{1}{3}\left(\frac{C_B}{C_K}\right)^4$$

where $$C_K = \sqrt{\frac{0.97 E K_{SE} K_T}{F_b}}$$

c) If $C_K < C_B \leq 50$, then

$$K_L = \frac{0.65 E K_{SE} K_T}{C_B^2 F_b K_X}$$

Where $F_b = f_b(K_D K_H K_{Sb} K_T)$

f_b = bending strength, MPa

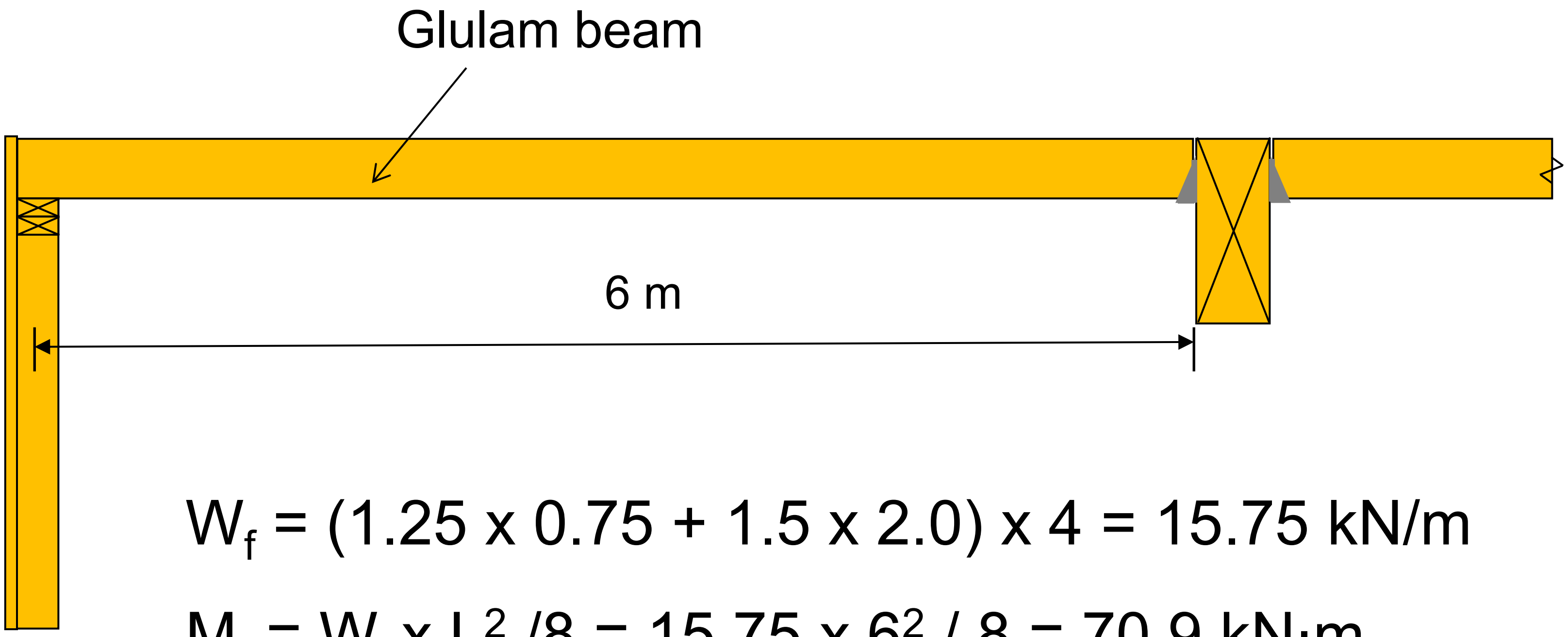
E = modulus of elasticity, MPa

K_X = Curvature Factor

Other K's are other modification factors

Ex 3 – Glulam beam design

D=0.75 kPa, S=2.0 kPa
Tributary width = 4m



$$W_f = (1.25 \times 0.75 + 1.5 \times 2.0) \times 4 = 15.75 \text{ kN/m}$$

$$M_f = W_f \times L^2 /8 = 15.75 \times 6^2 / 8 = 70.9 \text{ kN·m}$$

$K_X = K_D = K_H = K_{Sb} = K_T = K_{SE} = 1.0$

Ex 3 – Glulam beam design

Initial trial section:

- Species : Spruce-pine
- Grade : 20f-EX
- Size : 130mm x 380mm

$S = (130 \times 380^2)/6$

$= 3.12 \times 10^6 \text{ mm}^3$

Table 7.3
Specified strengths and modulus of elasticity for glued-laminated timber, MPa

	Spruce-Lodgepole Pine-Jack Pine			
	20f-E	20f-EX	14t-E	12c-E
Bending moment (pos.), f_b	25.6	25.6	24.3	9.8
Bending moment (neg.), f_b	19.2	25.6	24.3	9.8
Longitudinal shear, f_v	1.75	1.75	1.75	1.75
Compression parallel, f_c	25.2*	25.2*	25.2	25.2
Compression parallel combined with bending, f_{cb}	25.2*	25.2	25.2	25.2
Compression perpendicular, f_{cp} Compression face bearing	5.8	5.8	5.8	5.8
Tension face bearing	5.8	5.8	5.8	5.8
Tension net section, f_{tn} (see Clause 7.5.11)	17.0*	17.0	17.9	17.0
Tension gross section, f_{tg}	12.7*	12.7	13.4	12.7
Tension perpendicular to grain, f_{tp}	0.51	0.51	0.51	0.51
Modulus of elasticity, E	10 300	10 300	10 700	9 700

**The use of this stress grade for this primary application is not recommended.*

Ex 3 – Glulam beam design

7.5.6.5 Moment resistance

7.5.6.5.1

Except as provided for in Clauses 7.5.6.5.3 and 7.5.6.6, the factored bending moment resistance, M_r, of glued-laminated timber members shall be taken as the lesser of M_{r1} or M_{r2}, as follows:

$$M_{r1} = \phi F_b S K_x K_{Zbg}$$

$$M_{r2} = \phi F_b S K_x K_L$$

where

ϕ = 0.9

$F_b = f_b(K_D K_H K_{Sb} K_T)$

where

f_b = specified strength in bending, MPa (Table 7.3)

K_X = curvature factor (Clause 7.5.6.5.2)

$$K_{Zbg} = \left(\frac{130}{b}\right)^{\frac{1}{10}} \left(\frac{610}{d}\right)^{\frac{1}{10}} \left(\frac{9100}{L}\right)^{\frac{1}{10}} \leq 1.3$$

where

b = beam width (for single-piece laminations) or the width of widest piece (for multiple-piece laminations), mm

d = beam depth, mm

L = length of beam segment from point of zero moment to point of zero moment, mm

K_L = lateral stability factor (Clause 7.5.6.4)

Ex 3 – Glulam beam design

Volume factor, K_{Zbg} (7.5.6.5.1):

$K_{Zbg} = (130/130)^{1/10} \times (610/380)^{1/10} \times (9100/6000)^{1/10}$

$= 1.093$

Lateral stability factor, K_L (7.5.6.4.4):

$C_B = 16.09$

$l_u = 6000mm$,

$L_e = 1.92*6000=11520mm$

$$C_B = \sqrt{\frac{L_e d}{b^2}}$$

where

L_e = effective length, mm, from Table 7.5.6.4.3

$C_K = 19.76$

$$C_K = \sqrt{\frac{0.97 E K_{SE} K_T}{F_b}}$$

where

$F_b = f_b(K_D K_H K_{Sb} K_T)$

Ex 3 – Glulam beam design

Lateral stability factor, K_L (7.5.6.4.4):

$C_B = 16.09$, $C_K = 19.76$

Three possible conditions:

(a) when C_B does not exceed 10:

$K_L = 1.0$

(b) when C_B is greater than 10 but does not exceed C_K:

$$K_L = 1 - \frac{1}{3}\left(\frac{C_B}{C_K}\right)^4$$

→ $K_L = 0.85$

(c) when C_B is greater than C_K but does not exceed 50:

$$K_L = \frac{0.65 E K_{SE} K_T}{C_B^2 F_b K_X}$$

Ex 3 – Glulam beam design

$$M_{r1} = \phi\, F_b\, S\, K_X\, K_{Zbg}$$

$$M_{r1} = 0.9 \cdot 25.6 \cdot 3.12 \times 10^6 \cdot 1 \cdot 1.093$$
$$= 78.79 \text{ kN·m}$$

$$M_{r2} = \phi\, F_b\, S\, K_X\, K_L$$

$$M_{r2} = 0.9 \cdot 25.6 \cdot 3.12 \times 10^6 \cdot 1 \cdot 0.85$$
$$= 61.27 \text{ kN·m}$$

$M_f = 70.9$ kN·m, Not ok

Ex 3 – Glulam beam design

Remedy

Provide restraint to compression flange at 2m spacing,

- Unsupported length, a = 2000mm, L_e = 1.92*2000=3840mm
- $C_B = 9.29$
- $K_L = 1.0$, since $C_B \leq 10$

$$M_{r2} = 0.9 \cdot 25.6 \cdot 3.12 \times 10^6 \cdot 1 \cdot 1.0 = 71.88 \text{ kN}\cdot\text{m}$$

$M_f = 70.9$ kN·m, Ok

Shear Resistance § 7.5.7.2 (no notch)

For glulam beams of any volume, factored shear resistance, W_r > total factored loading, W_f, acting normal to a member

$$W_r = \phi F_v 0.48 A_g C_V Z^{-0.18} \geq W_f$$

All loads

where Φ=0.9

$F_v = f_v(K_D K_H K_{Sv} K_T)$

f_v= Specified strength in shear

A_g – Gross cross-section area

C_V – Shear load coefficient (§ 7.5.7.5)

Z – Beam volume, m^3

For volume < 2.0 m^3 – Simplified method (also applies to members other than beam), shear resistance of critical cross section, V_r

$$V_r = \phi F_v \frac{2A_g}{3}$$

Excluding loads within d of a support

Shear load coefficient, C_V

For most loading conditions, C_v, can be found in Tables 7.5.7.5A – 7.5.7.5F.

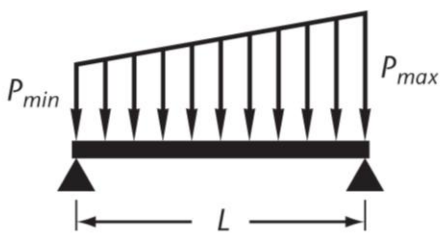
Table 7.5.7.5A
Shear load coefficient, C_V, for simple span beams

Number of equal loads equally and symmetrically spaced	r*			
	0.0	0.5	2.0	10.0 and over
1	3.69	3.34	2.92	2.46
2	3.69	3.37	3.01	2.67
3	3.69	3.41	3.12	2.84
4	3.69	3.45	3.21	2.97
5	3.69	3.48	3.28	3.08
6	3.69	3.51	3.34	3.16

$$*r = \frac{\textit{total of concentrated loads}}{\textit{total of uniform loads}}$$

Shear load coefficient, C_V

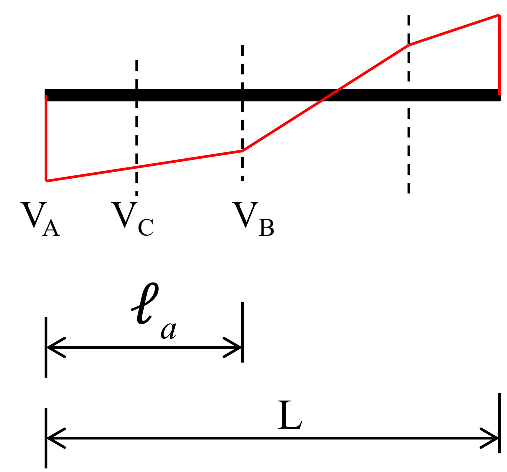
Table 7.5.7.5B
Shear load coefficient, C_V, for distributed loads

	P_{min}/P_{max}					
Type of loading	0.0	0.2	0.4	0.6	0.8	1.0
	3.40	3.55	3.63	3.67	3.69	3.69

Shear load coefficient, C_V

A general procedure is given in 7.5.7.5 for all cases:

1. Construct factored shear force diagram.
2. Divide into n segments, such that within each segment there are no abrupt changes nor changes from +ve to –ve values.
3. For each segment determine



(i) V_A = factored shear force at beginning of segment, N;
(ii) V_B = factored shear force at end of segment, N; and
(iii) V_C = factored shear force at centre of segment, N

and calculate the factor G as follows:

$$G = \ell_a \left[V_A^5 + V_B^5 + 4V_C^5 \right]$$

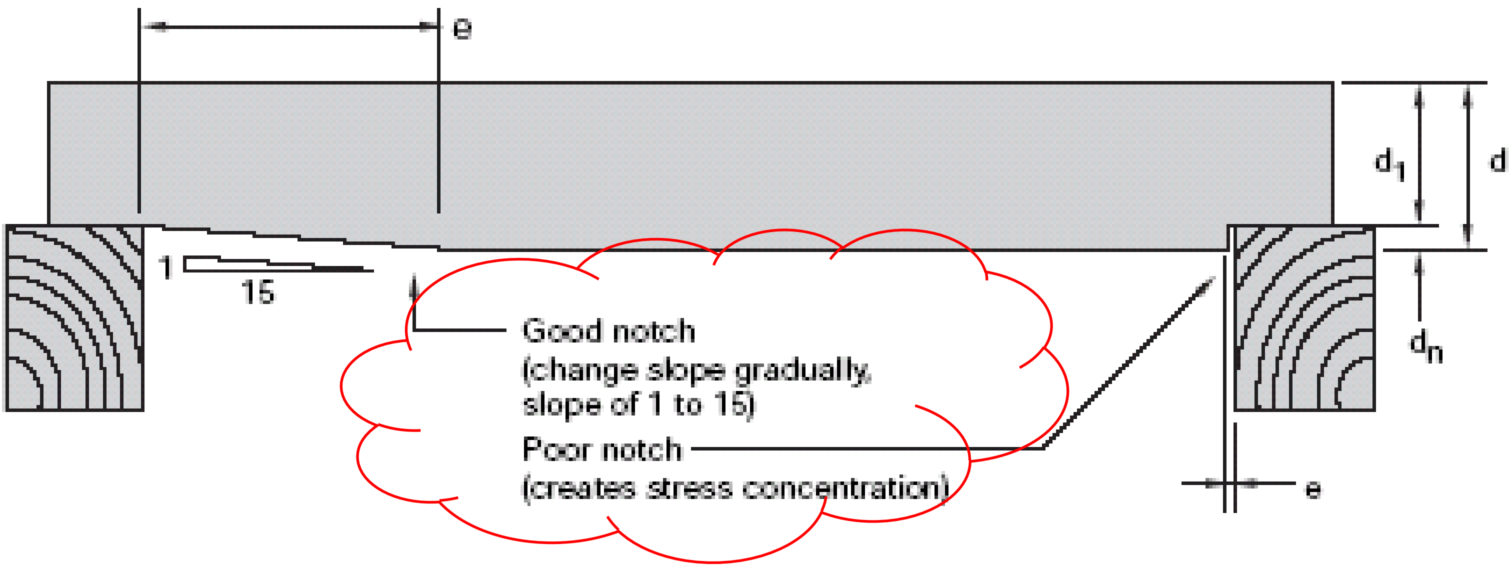
(All values are treated as +ve)

1. Determine C_v for the beam

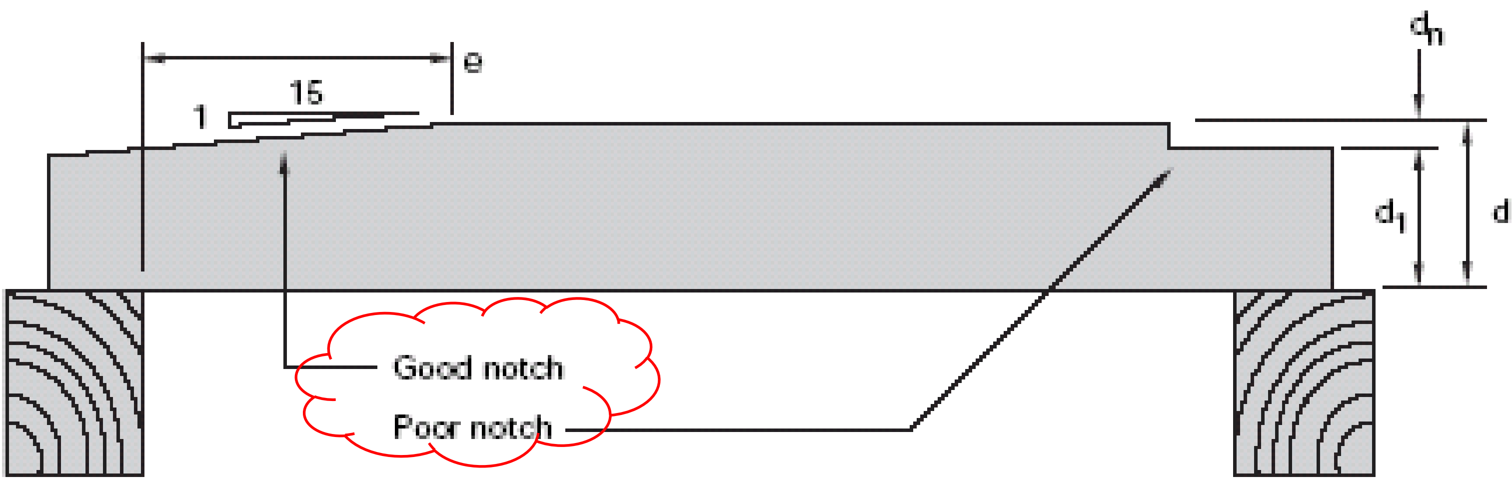
$$C_V = 1.825 W_f \left(\frac{L}{\Sigma G} \right)^{0.2}$$

where
W_f = the total of all factored moving loads and all factored distributed loads applied to the beam, N

Beam notched on lower face at the ends



Beam notched on upper face at the ends



Notch for Glulam - Compression side notch

Check shear resistance

(a) for $e_c > d$: $V_r = \phi F_v \frac{2A_n}{3}$

Net section below notch

(b) for $e_c < d$: $V_r = \phi F_v \frac{2A_g}{3}\left(1 - \frac{d_n e_c}{d(d - d_n)}\right)$

Gross section

where

$\phi = 0.9$

$F_v = f_v(K_D K_H K_{Sv} K_T)$

where

f_v = specified strength in shear, MPa (Table 7.3)

$A_n = b(d - d_n)$ = net cross-sectional area of member, mm^2 (Clause 7.5.4)

$A_g = b \times d$ = gross cross-sectional area of member, mm^2

b = member width, mm

d = member depth, mm

d_n = notch depth, mm (which shall not exceed $0.25d$)

e_c = length of notch, mm, from inner edge of closest support to farthest edge of notch

Notch for Glulam - Tension side notch

a) Check longitudinal shear resistance above notch (7.5.7.4.1) – shear capacity is not reduced by notch if notch is within d from support

b) Check factored fracture shear resistance, F_r, at notch according to 7.5.7.4.2 – identical to provision for sawn lumber except for f_f

$$F_r = \phi F_f A_g K_N$$

where

ϕ = 0.9

$F_f = f_f (K_D K_H K_{Sf} K_T)$

where

f_f = specified fracture shear strength at a notch, MPa
= $2.5\, b_{eff}^{-0.2}$ or 0.9 MPa, whichever is greater

where

b_{eff} = effective lamination width (mm)
= beam width (for single-piece laminations) or the width of widest piece (for multiple-piece laminations)

Other design requirements

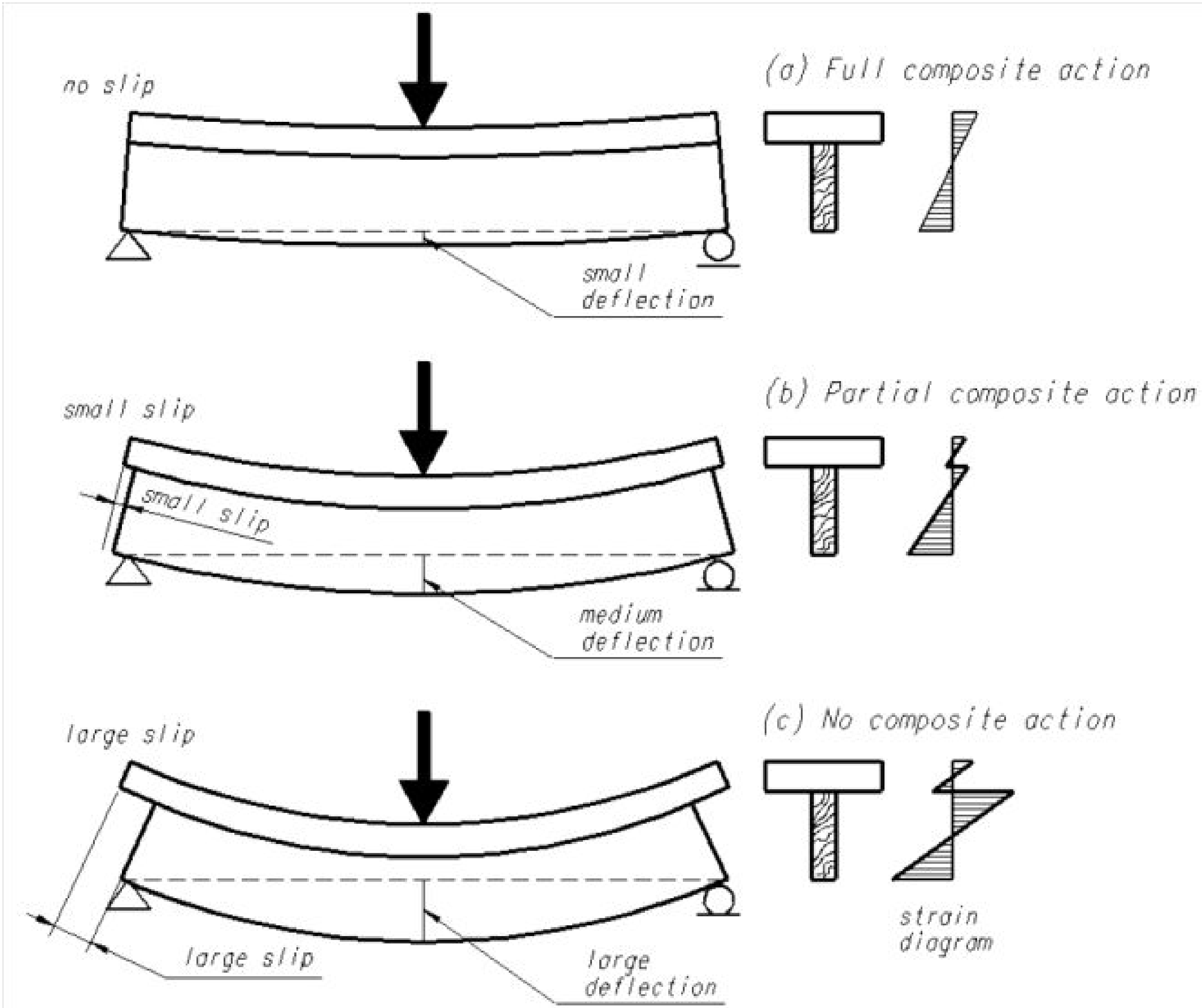
- Bearing check is similar to sawn lumber (7.5.9)
- Deflection requirements are similar to sawn lumber

Composite beam with semi-rigid connection – not explicitly considered in CSA O86

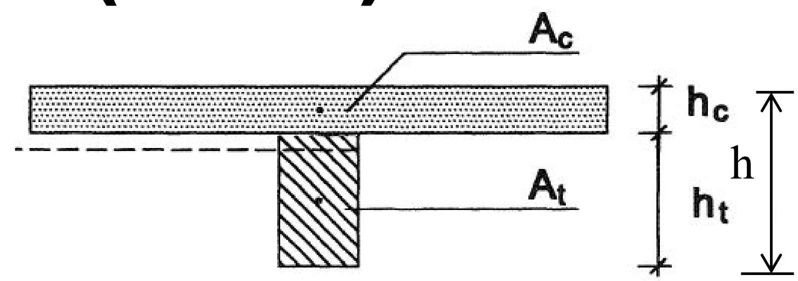
Applications

Two components connected by mechanical shear connectors

Behaviour of composite beam



Effective bending stiffness - Mohler's Model (1956)



$$EI_{\text{ef}} = E_{\text{t,d}}[I_{\text{tot}} + \gamma \cdot (nA_c e_c^2 + A_t e_t^2)]$$

The second moment of plane area I_{tot} is written as

$$I_{\text{tot}} = I_t + nI_c$$

and

$$\gamma = \frac{1}{1+p}$$

where

$$p = E_{\text{c,d}} \left(\frac{\pi}{l}\right)^2 \frac{1}{k} \frac{A_t A_c}{A_t + nA_c}$$

Span

n = modulus ratio

$$n = \frac{E_{\text{c,d}}}{E_{\text{t,d}}}$$

$$k = \frac{K}{s}$$

where
K = connection stiffness of 1 fastener, N/mm
s = connector spacing, mm
k = smeared connection stiffness, N/mm/mm

$$e_c = \frac{1}{2} h \frac{A_t}{A_t + nA_c}$$

$$e_t = \frac{1}{2} h \frac{nA_c}{A_t + nA_c}$$

Procedure:
1. Calculate n, A_c and A_t
2. Calculate p and γ
3. Calculate e_c and e_t
4. Calculate EI_{ef}

Mohler's Model - Assumptions

- The beam is simply supported.
- The components are connected by shear connectors, equally spaced, with a slip modulus (stiffness) of K.
- Spacing of connectors (s) is uniform along the beam.
- Loading is sinusoidal or parabolic.

  (Note : this allows a close-form solution to be obtained, hence solution is an approximation).

Composite beam – strength design

Note : EI_{ef} is dependent on span, since k is a function of span.

Extreme fibre stresses in components:

$$\sigma_c = \frac{0.5 E_{c,d}\, h_c\, M}{EI_{ef}} \quad , \quad \sigma_t = \frac{0.5 E_{t,d}\, h_t\, M}{EI_{ef}}$$

where M = bending moment at the cross section of interest

Shear force demand in a connector:

$$F_{con} = \frac{\gamma E_{c,d}\, e_c\, s V}{EI_{ef}}$$

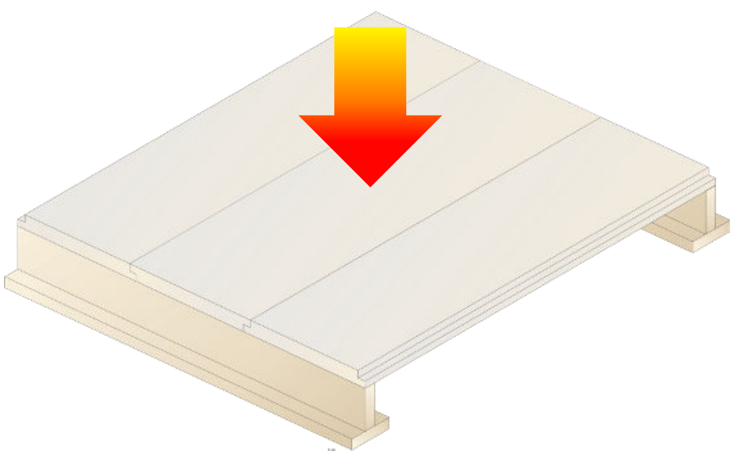
If F_{con} exceeds strength of a fastener, failure in shear connection will occur

where V = vertical shear force in the cross section of the connector

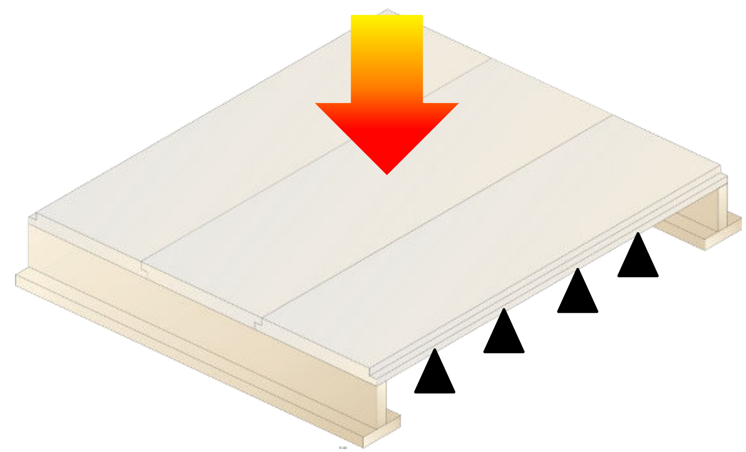
Cross laminated timber (CLT)

Bending properties – floor and roof panels

- Designs generally treat CLT as one-way beam and not two-way plate



One-dimensional beam
- Simple formula but conservative



Two-way plate
- More complex analysis, but accounts for real behaviour

CLT stress grades

8 Cross-laminated timber (CLT)

8.1 Scope

The design values and methods given in Clause 8 apply only to panels of primary and custom CLT stress grades manufactured and certified in accordance with ANSI/APA PRG 320 and layups as defined in Clause 8.2. Panels with alternative CLT layups shall be designed in accordance with Clause 4.3.2.

Table 8.2.3
Primary CLT stress grades

Stress grade	Species combinations and grades of laminations
E1	1950 F_b-1.7E Spruce-Pine-Fir MSR lumber in all longitudinal layers and No. 3/Stud Spruce-Pine-Fir lumber in all transverse layers
E2	1650 F_b-1.5E Douglas fir-Larch MSR lumber in all longitudinal layers and No. 3/Stud Douglas fir-Larch lumber in all transverse layers
E3	1200 F_b-1.2E Northern Species MSR lumber in all longitudinal layers and No. 3/Stud Northern Species lumber in all transverse layers
V1	No. 1/No. 2 Douglas fir-Larch lumber in all longitudinal layers and No. 3/Stud Douglas fir-Larch lumber in all transverse layers
V2	No. 1/No. 2 Spruce-Pine-Fir lumber in all longitudinal layers and No. 3/Stud Spruce-Pine-Fir lumber in all transverse layers

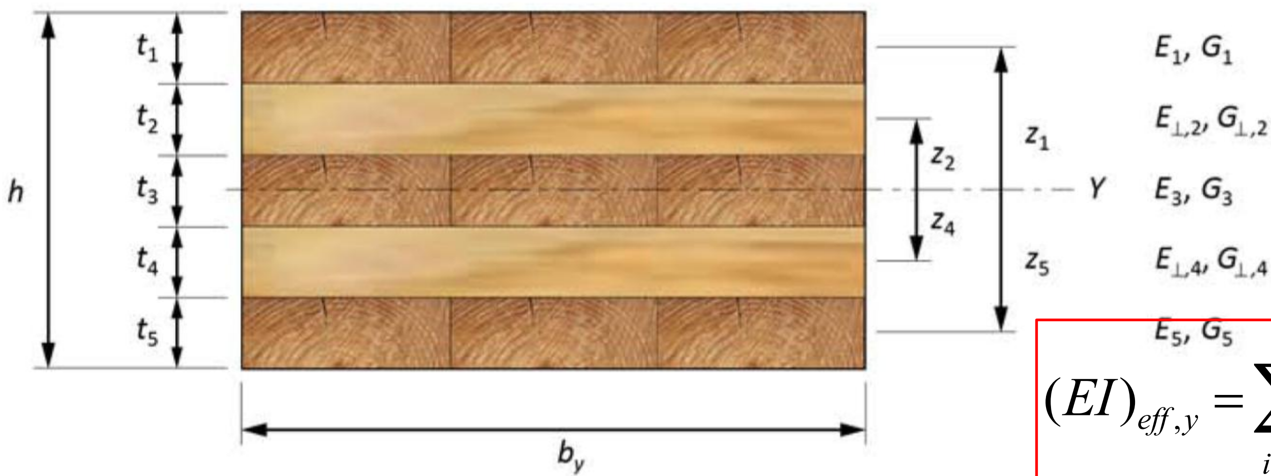
Lamination properties for calculation of CLT design properties

Table 8.2.4
Specified strengths and moduli of elasticity of laminations in primary CLT stress grades, MPa

Stress grade	Longitudinal layers						Transverse layers					
	f_b	E	f_t	f_c	f_s	f_{cp}	f_b	E	f_t	f_c	f_s	f_{cp}
E1	28.2	11700	15.4	19.3	0.50	5.3	7.0	9000	3.2	9.0	0.50	5.3
E2	23.9	10300	11.4	18.1	0.63	7.0	4.6	10000	2.1	7.3	0.63	7.0
E3	17.4	8300	6.7	15.1	0.43	3.5	4.5	6500	2.0	5.2	0.43	3.5
V1	10.0	11000	5.8	14.0	0.63	7.0	4.6	10000	2.1	7.3	0.63	7.0
V2	11.8	9500	5.5	11.5	0.50	5.3	7.0	9000	3.2	9.0	0.50	5.3

Notes:
Transverse modulus, $E_{\perp} = E/30$
$G = E/16$
Rolling shear modulus, $G_{\perp} = G/10$

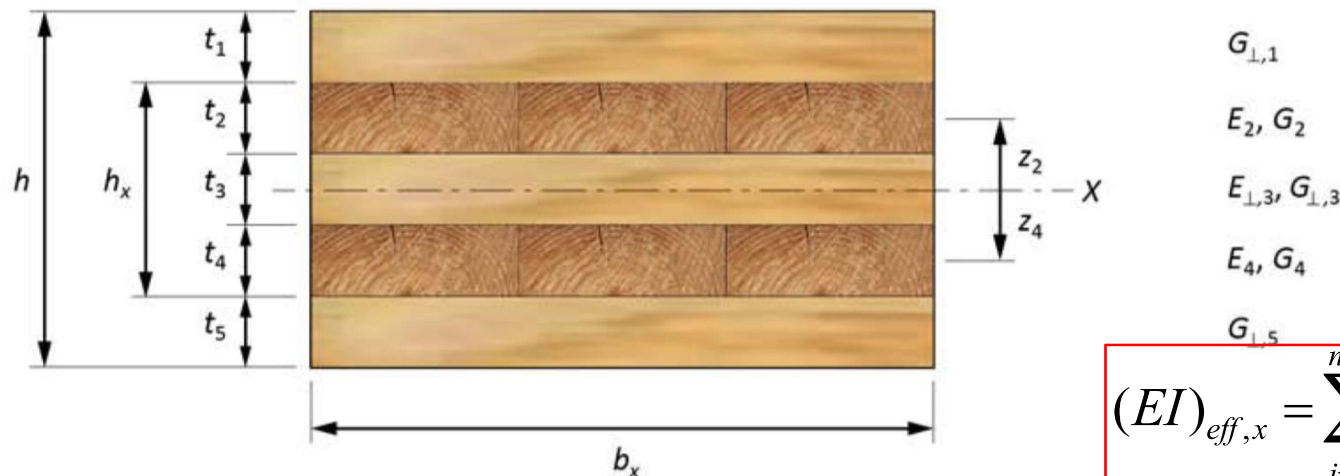
Effective Bending Stiffness



(a) Properties for the major strength axis

$$(EI)_{eff,y} = \sum_{i=1}^{n} E_i\, b_y \frac{t_i^3}{12} + \sum_{i=1}^{n} E_i\, b_y\, t_i\, z_i^2$$

where b_x, b_y are width of the panel



(b) Properties for the minor strength axis

$$(EI)_{eff,x} = \sum_{i=2}^{n-1} E_i\, b_x \frac{t_i^3}{12} + \sum_{i=2}^{n-1} E_i\, b_x\, t_i\, z_i^2$$

Surface layers are ignored

Bending moment resistance - Major

8.4.3.1 General

The out-of-plane factored bending moment resistance, M_r, of CLT panels shall be calculated as follows:

(a) for the major strength axis (Figure 8.4.3.2a):

$$M_{r,y} = \phi F_b S_{eff,y} K_{rb,y}$$

where

$\phi = 0.9$

$F_b = f_b(K_D K_H K_{Sb} K_T)$

where

f_b = specified bending strength of laminations in the longitudinal layers, MPa (Clause 8.2.4)

$$S_{eff,y} = \frac{(EI)_{eff,y}}{E} \frac{2}{h}$$

where

$(EI)_{eff,y}$ = effective bending stiffness of the panel for the major strength axis, N•mm^2 (Clause 8.4.3.2)

E = specified modulus of elasticity of laminations in the longitudinal layers, MPa (Clause 8.2.4, Figure 8.4.3.2a)

h = thickness of the panel, mm (Figure 8.4.3.2a)

$K_{rb,y} = 0.85$

Bending moment resistance - Minor

(b) for the minor strength axis (Figure 8.4.3.2b):

$$M_{r,x} = \phi F_b S_{eff,x} K_{rb,x}$$

where

ϕ = 0.9

F_b = $f_b(K_D K_H K_{Sb} K_T)$

where

f_b = specified bending strength of laminations in the transverse layers, MPa (Clause 8.2.4)

$$S_{eff,x} = \frac{(EI)_{eff,x}}{E} \frac{2}{h_x}$$

where

$(EI)_{eff,x}$ = effective bending stiffness of the panel for the minor strength axis, N•mm^2 (Clause 8.4.3.2)

E = specified modulus of elasticity of laminations in the transverse layers, MPa (Clause 8.2.4, Figure 8.4.3.2b)

h_x = thickness of the panel without the outer longitudinal layers, mm (Figure 8.4.3.2b)

$K_{rb,x} = 1.0$

Effective in-plane shear rigidity

$$(GA)_{eff,zy} = \frac{\left(h - \frac{t_1}{2} - \frac{t_n}{2}\right)^2}{\left[\left(\frac{t_1}{2\ G_1\ b_y}\right) + \left(\sum_{i=2}^{n-1} \frac{t_i}{G_i\ b_y}\right) + \left(\frac{t_n}{2\ G_n\ b_y}\right)\right]}$$

$$(GA)_{eff,zx} = \frac{\left(h - \frac{t_1}{2} - \frac{t_n}{2}\right)^2}{\left[\left(\frac{t_1}{2\ G_1\ b_x}\right) + \left(\sum_{i=2}^{n-1} \frac{t_i}{G_i\ b_x}\right) + \left(\frac{t_n}{2\ G_n\ b_x}\right)\right]}$$

where G is shear modulus of lamination
In minor direction, ignore surface layers.

Serviceability Limit States

8.5 Serviceability limit states

8.5.1 General

The design of CLT panels for serviceability limit states shall be in accordance with Clauses 5.1.3 and 5.4.

8.5.2 Deflection of CLT panels

The maximum deflection under a specified load acting perpendicular to the plane of the panel shall be calculated as a sum of the deflections due to moment and shear using the effective bending stiffness $(EI)_{eff}$, and the effective in-plane (planar) shear rigidity, $(GA)_{eff}$, as defined in Clause 8.4.3.2, with consideration for creep effects.

Note: *A method for calculating deflection under static uniform or concentrated load is provided in Clause A.8.5.2.*

8.5.3 Vibration performance of CLT floors

A method for calculating vibration-controlled spans for CLT floors is provided in Clause A.8.5.3.

Deflection calculation of CLT panel under transverse loads

Shear deflection and creep must be considered.

Under UDL for simply supported panel,

$$\Delta = \frac{5}{384}\frac{\omega L^4}{(EI)_{eff}} + \frac{1}{8}\frac{\omega L^2 \kappa}{(GA)_{eff}}$$

Under point load (line load across width) for simply supported panels,

$$\Delta = \frac{1}{48}\frac{PL^3}{(EI)_{eff}} + \frac{1}{4}\frac{PL\kappa}{(GA)_{eff}}$$

where

κ = form factor

= 1.0 (to be revised in future edition of CSA O86

Deflection calculation of CLT panel under transverse loads

Shear deflection and creep must be considered.

A.8.5.2 Deflection of CLT panels

The maximum deflection of the CLT panel, Δ_{max}, may be calculated as the sum of deflections under short and long term loads as follows:

$$\Delta_{max} = \Delta_{ST} + \Delta_{LT} K_{creep}$$

where

Δ_{ST} = elastic deflection due to short term and/or standard term loads, without dead loads in combination

Δ_{LT} = instantaneous elastic deflection due to long term loads

K_{creep} = creep adjustment factor

= 2.0 for dry service condition ← Magnify the dead load deflection by 100%

Note: *Where the shear deformation component of the total deformation of CLT panel under out of plane standard term loading such as snow and live loads is significant (i.e., in short spans, short span cantilever, etc.) as determined by the designer, the shear deformation under these loads should be increased by 30% to account for time-dependent effect associated with rolling shear. See the CWC Commentary on CSA O86 for more information.*

Vibration performance of CLT floor – A.8.5.3

Vibration-controlled span, l_v, is calculated from the following equation:

$$l_v \leq 0.11 \frac{\left(\frac{(EI)_{eff}}{10^6} \right)^{0.29}}{m^{0.12}}$$

where

l_v = vibration-controlled span limit, m

m = linear mass of CLT for a 1 m wide panel, kg/m

$(EI)_{eff}$ = effective bending stiffness for a 1m wide panel, N•mm^2 (see Clause 8.4.3.2)

For multiple-span floors with a non-structural element that is considered to provide enhanced vibration effect, the calculated vibration controlled span may be increased by up to 20%, provided it is not greater than 8 m.

End
Lecture #4

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