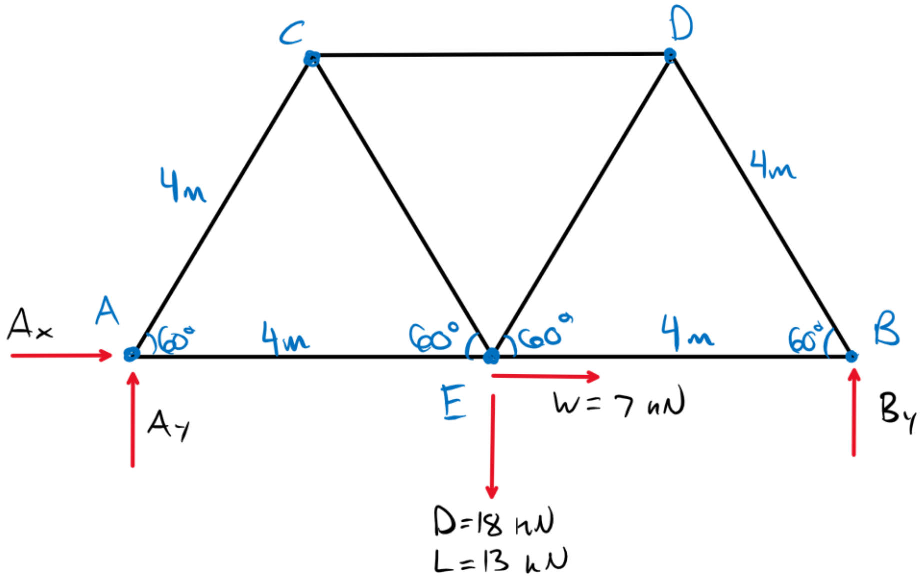
CWC Wood Engineering
Midterm Solution

QUESTION 1

a) First, we must use equilibrium to determine the relevant specified forces in truss member CE. To do this, we can create a section through members CD, CE, and AE.



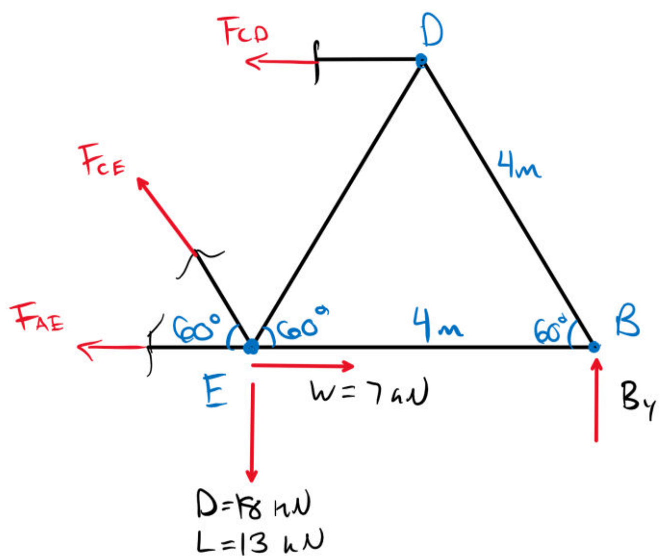
Using equilibrium, we can determine the reaction forces as follows:

$\Sigma F_x = 0 = A_x + W$ $\quad A_x = -W$

$\Sigma M_A = 0 = -4D - 4L + 8B_y$ $\quad B_y = (D+L)/2$

$\Sigma F_y = 0 = A_Y + B_y – D – L$ $\quad A_y = D + L – (D+L)/2 = (D+L)/2$

Now, taking a section:



Taking the sum of forces in the y-direction now gives:

$\Sigma F_y = 0 = F_{ce}\sin 60 - D - L + B_y$ $F_{ce} = (D+L-B_y)/\sin 60 = (D+L)/2\sin 60$

Member CE is in tension, so we can determine the governing load case by considering K_D:

Load Case 1: 1.4D $K_D = 0.65$

$P_{f,\,ce} = 14.5$ kN $P_f/K_D = 22.4$ kN

Load Case 2: 1.25D + 1.5L $K_D = 1 - 0.5\log(18/13) = 0.93$

$P_{f,\,ce} = 24.2$ kN $P_f/K_D = 26.1$ kN **(GOVERNS tensions in member CE)**

b) We must design a Nothern No. 1 tension member to resist a factored load of 24.2 kN with $K_D = 0.93$. The net tension requirement can be approximated as follows by also accounting for the 25% gross section reduction:

$P_f/0.75K_D = 34.7$ kN

From the selection tables, try Nothern No.1 38x235 ($P_r = 35.4$ kN). Now perform the design check.

For structural joists and planks (**Table 6.2.2.1**)

$f_t = 4.0$ MPa
$F_t = f_t K_D K_H K_s K_t = (4.0\text{ MPa})(0.93)(1)(1)(1) = 3.72$ MPa

$K_{zt} = 1.1$ (**Table 6.4.5**)

For lumber in tension:

$T_r = \phi F_t A_n K_{zt} = (0.90)(3.72\text{ MPa})(0.75 \times 38\text{x}325\text{ mm}^2)(1.1)$
$T_r = 24.7\text{ kN} > T_f = 24.2\text{ kN}$ **(Section Passes Tension Check)**

Therefore, a Nothern No.1 38x325 mm member is suitable and achieves a utilization of 98% (T_f/T_r).

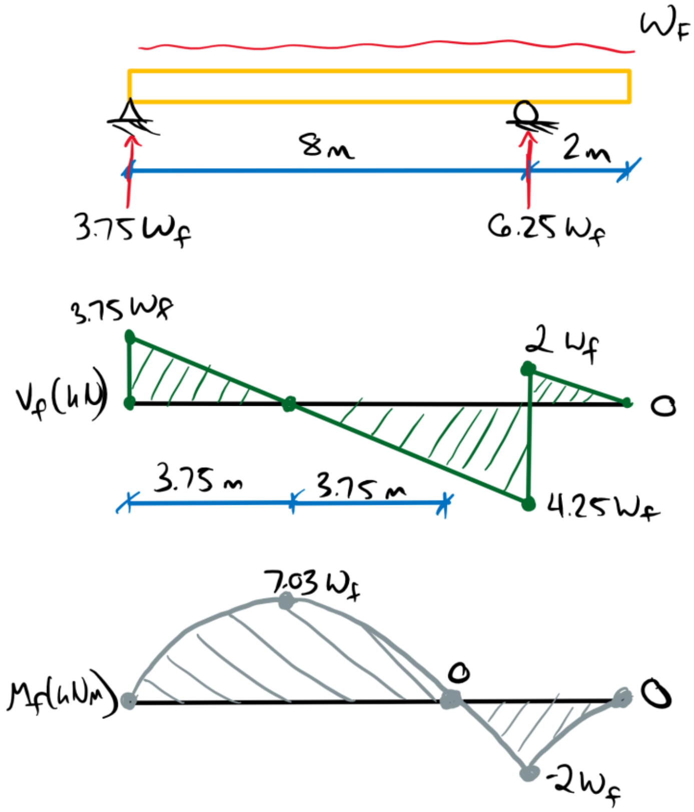
b) By inspection, member BD is in compression. Per **6.5.6.2.2**, the slenderness ratio must be below 50.

$C_{cx} = L_e/d = 4000/325 = 12.3 < 50$
$C_{cy} = 4000/38 = 105 > 50$ **(Section too slender for compression – cannot be used)**

Therefore, the member used for CE is too slender for BD and cannot be used.

QUESTION 2

The question requires the calculation of the critical factored load on a cantilevered SPF 20f-E 315x760 mm member in wet service conditions. Creating the beam diagrams gives the following factored loads:



$V_f = 4.25w_f$
$W_f = 10w_f$
$M_{f,+} = 7.03w_f$
$M_{f,-} = -2w_f$

For SPF 20f-E

$f_{b,+} = 25.6$ MPa
$f_{b,-} = 19.2$ MPa
$f_v = 1.75$ MPa
$E = 10300$ MPa

<u>Shear Resistance (**7.5.7**):</u>

$Z = bxdxL = (0.315 \times 0.760 \times 10\ m^3) = 2.39\ m^3 > 2.0\ m^3$

Therefore, **7.5.7.2a** applies. From the shear force diagram, we can establish three segments per **7.5.7.5**.

Segment 1: $G_1 = (3.75m)((3.75w_f)^5+0+4(1.875w_f)^5) = 3128.5w_f^5$
Segment 2: $G_2 = (4.25m)(0+(4.25w_f)^5+4(2.125w_f)^5) = 6629.6w_f^5$

Segment 3: $G_3 = (2m)((2w_f)^5+0+4(1w_f)^5) = 72w_f^5$
$C_v = 1.825(10w_f) (10 / 9830.1w_f^5)^{1/5} = 4.6$

Alternatively, we can interpolate C_v from **Table 7.5.7.5C** with:

$r^*= 0/w_f = 0$ $L_1/L_2 = 2/8 = 0.25$
$C_v = (4.55+4.88)/2 = 4.71$ (relatively close to 4.6 determined above).

$W_r = \phi F_v 0.48 A_g C_v Z^{-0.18}$

$F_v = f_v K_D K_H K_{sv} K_T = (1.75 \text{ MPa})(1)(1)(0.87)(1) = 1.52 \text{ MPa}$

$W_r = (0.90)0.48(760x315 \text{ mm}^2)(4.71)(2.39 \text{ m}^3)^{-0.18}$
$W_r = 416 \text{ kN} \geq W_f = 10w_f$
$w_f \leq 41.6 \text{ kNm}$ **(GOVERNS - Critical factored load for shear resistance)**

Moment Resistance (**7.5.6.5**):
There is a point of inflection in the applied moment at the roller support which must be treated per the note in **7.5.6.5**.

$K_{zbg,\,1} = (130 \text{ x } 610 \text{ x } 9100/ 157.5 \text{ x } 760 \text{ x } 7500)^{1/10} = 0.98 \leq 1.3$
$K_{zbg,\,2} = 1.1 \leq 1.3$

Assuming the beam is supported continuously:

$K_L = 1.0$

$S = bd^2/6 = 30.32 \text{ x } 10^6 \text{ mm}^3$

$F_{b,+} = f_{b,+} K_D K_H K_{sb} K_t = (25.6 \text{ MPa})(1)(1)(0.80)(1) = 20.5 \text{ MPa}$
$F_{b,-} = (19.2 \text{ MPa})(1)(1)(0.80)(1) = 15.4 \text{ MPa}$

The relevant M_r is calculated for the factored moment in that segment.

$M_{r1,\,1} = \phi F_{b,+} S K_x K_{zbg,\,1} = (0.90)(20.5 \text{ MPa})(30.32 \text{ x } 10^6 \text{ mm}^3)(1)(0.98)$
$M_{r1,\,1} = 548 \text{ kNm} \geq M_{f,\,1} = 7.03w_f$
$w_f \leq 78 \text{ kNm}$ **(Critical factored load for moment resistance in segment 1)**

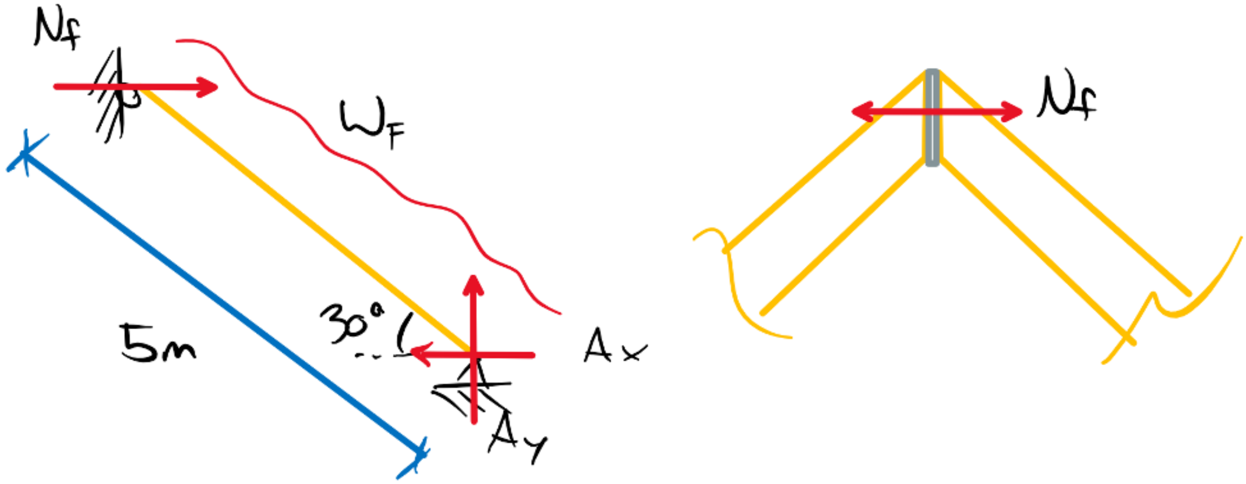
$M_{r1,\,2} = \phi F_{b,-} S K_x K_{zbg,\,2} = (0.90)(15.4 \text{ MPa})(30.32 \text{ x } 10^6 \text{ mm}^3)(1)(1.1)$
$M_{r1,\,2} = 462 \text{ kNm} \geq M_{f,\,2} = 2w_f$
$w_f \leq 231 \text{ kNm}$ **(Critical factored load for moment resistance in segment 2)**

The moment resistance based on K_L is calculated for the full length of the beam but, by inspection, won't govern over $M_{r1,1}$ and need not be checked.

Therefore, the critical factored load for this beam is 41.6 kNm based on the shear resistance of the member.

QUESTION 3

This question requires the determination of the maximum factored distributed load that can be applied based on the bearing capacity at an angle to the grain. With equilibrium, we can determine the factored bearing load relative to the distributed load.



$\Sigma M_A = 0 = w_f(5\text{ m})(2.5\text{ m}) - N_f(5\text{ m})\sin 30$

$N_f = 5w_f$

For D. Fir-L No.1, 143x343 mm:

$f_c = 11.0$ MPa

$f_{cp} = 7.0$ MPa

$E_{05} = 8000$ MPa

$E = 12000$ MPa

Bearing Capacity at an Angle to Grain (**6.5.8**):

Bearing at an angle to grain requires a summation of axial compression and bearing resistances.

$N_r = P_rQ_r/(P_r\sin^2\theta + Q_r\cos^2\theta)$

Axial Compression Resistance (**6.5.6.2.3**):

$P_r = \phi F_cAK_{zc}K_c$

$K_L = 1.0$ (Per **6.5.8**)

$F_c = f_cK_DK_HK_{sc}K_T = 11.0\text{ MPa }(1)(1)(1)(1) = 11.0$ MPa

$K_{zc} = 6.3(343 \times 5000\text{ mm}^2)^{-0.13} = 0.97 \leq 1.3$

$P_r = (0.80)(11.0\text{ MPa})(343\text{x}140\text{ mm}^2)(0.97)(1.0)$

$P_r = 410$ kN

Bearing Resistance (**6.5.7.2**):

$Q_r = \phi F_{cp} A_b K_B K_{zcp}$

$F_{cp} = f_{cp} K_D K_H K_{sc} K_T$ = (7.0 MPa)(1x1x1x1) = 7.0 MPa
K_{zcp} = 1.0 (b/d < 1.0 per **6.5.7.4**)
K_B = 1.0 (near end of member **6.5.7.4**)

Q_r = (0.80)(7.0 MPa)(140x343/cos30 mm^2)(1)(1)
Q_r = 310 kN

For an angle θ to the grain:

N_r = (410 kN)(310 kN)/(410sin^{2}30 + 310cos^{2}30)
N_r = 379 kN ≥ N_f = 5w_f
w_f ≤ 75.8 kNm **(Critical factored load for bearing capacity)**

b) This critical load will likely never govern as the moment and shear capacity limit more critical factored loads. Approximating the beam as simply supported (ignoring axial + bending), the following critical loads for shear and bending are determined from the tabulated resistance values for a 140x343 No.1 D. Fir-L member.

Shear:

V_r = 43.2 kN ≥ V_f = $w_f L/2$
w_f ≤ 17.3 kN/m

Moment:

M_r = 39.0 kNm ≥ M_f = $w_f L^2/8$
w_f ≤ 12.5 kN/m

Therefore, it is unlikely the bearing capacity will govern before shear or moment failure occurs.